$$\operatorname{Im} M^{(0)}_{\alpha\beta} = \frac{1}{2\pi} \sum_{\xi} k_{\xi} m_{\xi} M^{(0)}_{\alpha\xi} M^{(0)*}_{\beta\xi} + \frac{1}{4} \rho_c M_{\alpha c} M^{*}_{\beta c}. \qquad (9)$$

Expression (7) is obtained from (9) by substituting in the latter formulas (5) and (3), subject to the additional condition

$$\gamma^2 \rho_c \Gamma_0 << 1, \qquad (10)$$

which is equivalent (recognizing that $\gamma^2 \sim \Gamma_c/\Gamma_0$) to the requirement that the narrow levels be isolated:

$$\Gamma_c \rho_c << 1. \qquad (11)$$

Formulas (6) and (7) can be verified experimentally (in particular, using giant analog resonances as an example; we note that a formula similar to (7) was obtained for this case in [3] by another method).

The foregoing analysis is not connected with any dynamic model. It shows that the physical cause of the appearance of the fine structure of giant resonance is essentially the resonant rescattering in the initial and final states.

The author is grateful to B. V. Geshkenbein, D. F. Zaretskii, B. L. Ioffe, A. S. Kudryavtsev, K. A. Ter-Martirosyan, and M. G. Urin for useful discussions.

[1] H. Feshbach, Congres International de Physique Nucleaire, Paris, Juillet, 1, 227 (1964).
[2] C. Bloch, ibid. 1, 227 (1964) [sic!].
[3] D. Robson, Phys. Rev. 137, B535 (1965).

## OSCILLATIONS OF ABSORPTION AND AMPLIFICATION OF ULTRASOUND IN INELASTIC SCATTERING OF ELECTRONS

V. I. Mel'nikov
Institute of Theoretical Physics, USSR Academy of Sciences
Submitted 12 June 1968
ZhETF Pis. Red. 8, No. 3, 162-164 (5 August 1968)

Singularities of galvanomagnetic phenomena in strong electric fields, in the case of inelastic scattering of electrons with emission of optical phonons, were predicted in [1].

The same scattering mechanism can lead to oscillations of absorption and amplification of sound in a many-valley semiconductor as a function of the sound frequency $\omega$ or the electric field intensity E.

Let us consider a semiconductor with two valleys that shift in opposite directions upon passage of a sound wave. The electric field E and the sound wave vector q are parallel and are directed symetrically with respect to the valleys. The numerical factors governed by the geometry of the valleys will be written out for the case of n-Ge with sound propagating along a fourfold axis. We choose as the sole scattering mechanism the emission by the electron of an intervalley phonon with energy $\hbar\omega_0$ at the instant when this is allowed by the energy conservation law. During all the remaining time the electron moves, without scattering, under the influence of the electric field and the deformation potential, along a straight line in p-space, parallel to E and beginning at the minimum-energy point at which the electron lands after scattering. The criterial of such an approximation were discussed in [1].

Introducing the electron densities per unit length in p-space, $n_1(p)$ and $n_2(p)$ for the first and second valleys respectively, we write down the equation for collisionless motion:

$$\frac{\partial n_\alpha}{\partial t} + v\frac{\partial n_\alpha}{\partial r} + (eE - iq\delta\epsilon_\alpha)\frac{\partial n_\alpha}{\partial p} = 0; (\alpha = 1, 2), \qquad (1)$$

where $\delta\varepsilon_\alpha \sim \exp[-i\omega t + iqr]$ is the shift of the bottom of the $\alpha$th valley upon deformation, $v = p/m_\parallel$ and $m_\parallel$ are the velocity and mass of the electron in the direction of E and q. Phonon emission takes place when the electron momentum reaches the value

$$p_{0\alpha} = \sqrt{2m_\parallel(\hbar\omega_0 - 2\delta\epsilon_\alpha)} \approx p_0\left[1 - \frac{\delta\epsilon_\alpha}{\hbar\omega_0}\right]; p_0 = \sqrt{2m_\parallel\hbar\omega_0}. \qquad (2)$$

We took into account here the fact that $\delta\varepsilon_1 = -\delta\varepsilon_2$.

The solution of (1), accurate to terms linear in $\delta\varepsilon$, is

$$n_\alpha(p) = \frac{n}{p_0}\theta(p_{0\alpha} - p)\theta(p)\left\{1 + C\delta\epsilon_\alpha\exp\left[i\left(\omega - \frac{qp}{2m_\parallel}\right)\frac{p}{eE}\right]\right\}. \qquad (3)$$

where n is the electron density in one valley.

The product of the step functions shows clearly the region ($0 < p < p_{0\alpha}$) in which the electron can move.

The constant C should be determined from the boundary condition

$$(eE - iq\delta\epsilon_1)n_1(0) = (eE - iq\delta\epsilon_2 - \frac{\partial p_{02}}{\partial t}\; n_2(p_{02}), \qquad (4)$$

which corresponds to conservation of the number of electrons in scattering. To the same degree of accuracy, we get from (3) and (4)

$$C = \frac{i}{eE}\left(2q + \frac{p_0\omega}{\hbar\omega_0}\right)\left\{1 + \exp\left[i\left(\omega - \frac{qp_0}{2m_\parallel}\right)\frac{p_0}{eE}\right]\right\}^{-1}. \qquad (5)$$

In view of the fact that the speed of sound $w = \omega/q$ is much smaller than the electron velocity $p_0/m_\parallel$, we can neglect $\omega$ in (3) and (5) compared with $qp/2m_\parallel$:

$$n_\alpha(p) = \frac{n}{p_0}\theta(p_{0\alpha} - p)\theta(p)\left\{1 + \frac{i\delta\epsilon_\alpha}{eE}\cdot\frac{2q\exp\left(-\frac{iqp^2}{2m_\parallel eE}\right)}{\left[1 + \exp\left(-\frac{iqp_0^2}{2m_\parallel eE}\right)\right]}\right\}. \qquad (6)$$

Using formulas (2.11) and (2.12) of [2] to determine the sound absorption coefficient $\Gamma$ in terms of

$$\delta n_\alpha = \int_0^{p_{0\alpha}} n_\alpha(p)dp - n,$$

we obtain as the final result

$$\Gamma = \Gamma_M \frac{4kT}{\hbar\omega_0}\left\{1+\sqrt{\frac{\pi Y}{2}}\left[C(\sqrt{Y})\,\mathrm{tg}\,\frac{Y}{2} - S(\sqrt{Y})\right]\right\}. \tag{7}$$

Here C(x) and S(x) are the Fresnel integrals, $\Gamma_M$ is defined in [2], and

$$Y = \frac{qp_0^2}{2m_\parallel eE} = \frac{q\hbar\omega_0}{eE} = 2\pi\frac{\hbar\omega_0}{eE\lambda} = 2\pi\frac{l}{\lambda}. \tag{8}$$

It is seen from (7) and (8) that Γ depends in critical manner on the ratio of the electron mean free path $\ell = \hbar\omega_0/eE$ to the wavelength of the sound $\lambda = 2\pi w/\omega$, so that when $\ell = (n + 1/2)\lambda$ (where n is an arbitrary integer) Γ changes from +∞ to −∞. A more realistic calculation will smooth out these discontinuities, byt so long as the electron mean free path has a scatter that is smaller than the wavelength of sound, geometric resonance should be observed at these lengths. We emphasize that in this case Γ oscillates with reversal of sign, i.e., the sound is amplified or absorbed, depending on the ratio of ℓ and λ.

The author is grateful to E. I. Rashba for valuable discussions.

[1] I. I. Vosilius and I. B. Levinson, Zh. Eksp. Teor. Fiz. 50, 1660 (1966) and 52, 1013 (1967) [Sov. Phys.-JETP 23, 1104 (1966) and 25, 672 (1967)]; ZhETF Pis. Red. 6, 854 (1967) [JETP Lett. 6, 295 (1967)].
[2] S. V. Gantsevicn ahd V. L. Gurevich, Phys. Rev. 161, 736 (1967).

## E R R A T A

Article by E. M. Epshtein, Vol. 7, No. 11

In formula (5) on p. 341 and in the line following it, "$I_n$" should be replaced by "$J_n$."

On p. 342, lines 16-17, "given sound wave 2π/q" should be replaced by "sound wavelength 2π/q."

Article by L. N. Pyatnitskii et al., Vol.7, No. 12

Formula (1) on p. 378 is in error. The corrected formula reads

$$\Gamma_a(x) = e^{-x^2}\{[1 - a^2(2xe^{-x^2}\int_0^x e^{t^2}dt - 1]^2 + \pi a^4 x^2 e^{-2x^2}\}^{-1}.$$