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OCCURRENCE OF STRONG DISCONTINUITIES IN A PLASMA MOVING IN A MAGNETIC FIELD

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The drift of quasineutral inhomogeneities of electron and ion density in a plasma situated in a magnetic field was investigated sufficiently thoroughly in the linear approximation [1]. The purpose of the present paper is to consider simple nonlinear problems. We assume that the inhomogeneity is one-dimensional, i.e., it depeonds only on one spatial variable x, and that its dimension is sufficiently large; this allows us to neglect diffusion processes. Then, using (2.1) of [1] and eliminating from these equations the electric field, we get

$$\frac{\partial N}{\partial t} + \frac{\partial}{\partial x}[NV(N)] = 0, \tag{1}$$

$$V(N) = \frac{(\sigma_{\parallel e}\cos^2\beta + \sigma_{\perp e}\sin^2\beta)v_{ix} + (\sigma_{\parallel i}\cos^2\beta + \sigma_{\perp i}\sin^2\beta)v_{ex}}{(\sigma_{\parallel e} + \sigma_{\parallel i})\cos^2\beta + (\sigma_{\perp e} + \sigma_{\perp i})\sin^2\beta}. \tag{2}$$

Here β is the angle between the x axis and the magnetic field $\vec{H}$; $\sigma_{\parallel e}$, $\sigma_{\parallel i}$, $\sigma_{\perp e}$, and $\sigma_{\perp i}$ are the longitudinal and transverse components of the conductivity tensors for the electrons and ions; v_{ex} and v_{ix} are the x-projections of the electron and ion drift velocities in the homogeneous plasma. Equation (1) can also be rewritten in the form

$$\frac{\partial N}{\partial t} + V_{ax}(N)\frac{\partial N}{\partial x} = 0, \quad V_{ax}(N) = \frac{d}{dN}[NV(N)]. \tag{3}$$

Here V_{ax} is the x-projection of the ambipolar-drift velocity[1)].

The general solution of (3) can be written in the implicit form

$$N = N_0(x - V_{ax}(N)t), \tag{4}$$

where $N_0(x_0)$ is a function describing the distribution of the plasma density at the initial instant of time. Owing to dependence of the velocity V_{ax} on N, the initial distribution becomes deformed in the course of time, in analogy with the case of simple Riemann waves in

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1) We note an inaccuracy in formula (2.8) of [1] for the velocity V_a. This formula is rigorously valid only for the case of a weakly ionized plasma. In the general case we have

$$V_a = \frac{\partial}{\partial N_0}\left[N_0\frac{(\sigma_{\parallel e}\cos^2\beta + \sigma_{\perp e}\sin^2\beta)v_{i0} + (\sigma_{\parallel i}\cos^2\beta + \sigma_{\perp i}\sin^2\beta)v_{e0}}{(\sigma_{\parallel e} + \sigma_{\parallel i})\cos^2\beta + (\sigma_{\perp e} + \sigma_{\perp i})\sin^2\beta}\right].$$

Appropriate changes must also be made in formulas (2.9) and (2.4).

one-dimensional gas flow [2]. At a certain instant of time the wave bends over and a strong discontinuity arises in the electron and ion density distribution. The instant of formation of the discontinuity t_c and the value of N_c at the location of the discontinuity formation are defined by the conditions:

$$\frac{\partial x}{\partial N} = 0, \quad \frac{\partial^2 x}{\partial N^2} = 0$$

or

$$\left(\frac{dV_{ax}}{dN} \frac{d^2 x_0}{dN^2}\right)_{N=N_c} = \left(\frac{d^2 V_{ax}}{dN^2} \frac{dx_0}{dN}\right)_{N=N_c};$$

$$t_c = -\left(\frac{dx_0/dN}{dV_{ax}/dN}\right)_{N=N_c}. \tag{5}$$

Here $x_0(N)$ is a function inverse to $N_0(x_0)$, and the function $V_{ax}(N)$ is determined by formulas (2) and (3).

The discontinuity velocity V_p is determined from the condition of the conservation of the particle flux through the discontinuity surface. This yields

$$V_p = (N_1 V_1 - N_2 V_2)/(N_1 - N_2). \tag{6}$$

Here N_1 and N_2 are the concentrations to the left and right of the discontinuity, and $V_1 = V(N_1)$ and $V_2 = V(N_2)$ are the velocities defined by formula (2). In the case of a weak discontinuity, $(N_1 - N_2) \to 0$, the velocity V_p coincides with the linear drift velocity V_{ax}, as should be the case. The values of the concentrations N_1 and N_2 at each instant of time are determined by (4) (upper and lower solution) under the additional condition that the total number of particles must be conserved:

$$\int N(x, t)\,dx = \int N_0(x_0)\,dx_0.$$

When determining the structure of the discontinuity front, it is necessary to take into account the diffusion terms. From (2.1) of [1] we get

$$D_a(N) \frac{dN}{dx} = N[V(N) - V_p] - N_2(V_2 - V_p) = N_1(V_1 - V_p) - N[V_p - V(N)]. \tag{7}$$

Here $z = x - V_p t$, and $D_a(N)$ is the coefficient of ambipolar diffusion ([1], formula (1.12)). From this we determine in implicit form the dependence of N on z by direct integration. We see that the width of the discontinuity front is

$$L = \frac{D_a(N_p)}{N_1 |V_1 - V_p|} = \frac{D_a(N_p)}{N_2 | V_2 - V_p|}. \tag{8}$$

Here N_p is the value of the concentration at which $V(N_p) = V_p$. As usual, L increases with decreasing size of the discontinuity.

Let us consider by way of an example the drift of the plasma in crossed electric and magnetic fields, $\vec{E} \perp \vec{H}$. Let there be an inhomogeneity in the direction $x \parallel \vec{E} \times \vec{H}$. We then obtain from (2) for the velocity V(N)

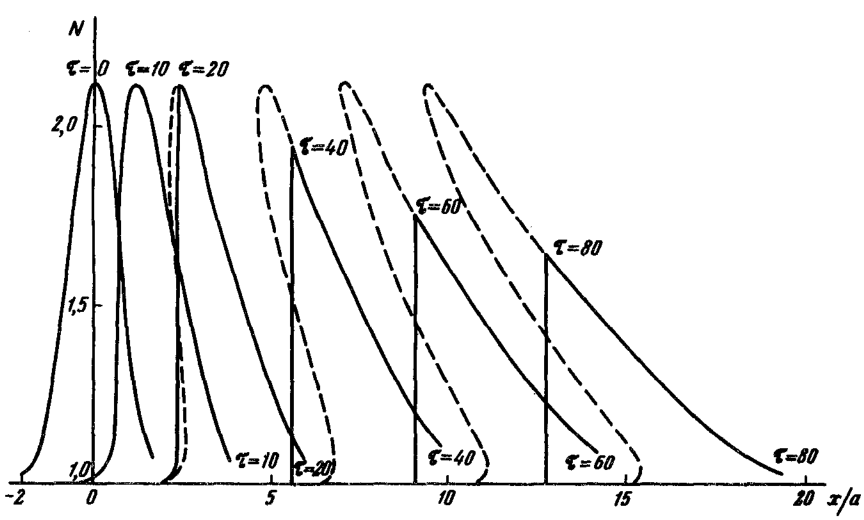
$$V(N) = \frac{eE\Omega_H}{M} \times$$

$$\times \frac{\omega_H^3\Omega_H + 2\omega_H^2\nu_{el}\nu_{im} + \frac{M}{m}\omega_H^2\nu_{im}^2 + (\nu_{el}+\nu_{em})(\nu_{em}^2\nu_{im} + \frac{M}{m}\nu_{el}\nu_{im}^2)}{[\nu_{im}(\nu_{el}+\nu_{em}) + \omega_H\Omega_H][(\nu_{em}+\nu_{el})^2\nu_{im}^2 + \omega_H^2(\nu_{im}^2 + 2\frac{m}{M}\nu_{el}\nu_{im} + \Omega_H^2)]}. \quad (9)$$

Here ν_{el} is the frequency of the collisions of the electrons with the ions and is proportional to N; ν_{em} and ν_{im} are the frequencies of the collisions with the neutral atoms (molecules), and ω_H and Ω_H are the gyrofrequencies for the electrons and the ions. In a weakly-ionized plasma, when the collision frequency ν_{el} can be neglected, the velocity V does not depend on N and the form of the inhomogeneity does not become distorted: its motion is always linear. To the contrary, at sufficiently high degrees of ionization, the dependence of V on N can be quite appreciable. For example, when $\nu_{el}(N_0) = \nu_{em}$ and $\nu_{im}\nu_{em} = \omega_H\Omega_H$, we get from (9) (when $N/N_0 \ll \sqrt{M/m}$):

$$V(N) = c\frac{E}{H}\frac{1}{2 + N/N_0}; \quad V_{ax} = c\frac{E}{H}\frac{2}{(2 + N/N_0)^2}. \quad (10)$$

The time variation of the inhomogeneity is shown for this case in the figure. The solid curves correspond to different instants of the time $t = (a/V_{N=N_0})\tau$ (a = characteristic dimension). As seen from the figure, a discontinuity is formed at the instant of time $t = 12a/V_{N=N_0}$. The magnitude of the discontinuity first increases rapidly, and then de-



creases with increasing t. The entire inhomogeneity spreads out in the course of time as a result of the nonlinear dispersion of the drift velocity. Nonlinear dissipation of the inhomogeneity is observed.

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USE OF IMAGES OF VELOCITY SPACE TO STUDY THE KINEMATICS OF π^-p INTERACTIONS AT 7.5 GeV WITH IDENTIFIED PROTON

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Using the new method of analyzing kinematics in images of Lobachevskii velocity space [1], proposed by one of the authors (Bubelev) [2], we analyzed anew 300 of the ~1300 4-prong π^-p interactions in propane at energy 7.5 ± 0.6 GeV, which were investigated in [3], namely, events with the proton identified by ionization ($p_p \lesssim 1$ GeV/c) from [3a].

We used in the method the longitudinal L-diagram of Lobachevskii velocities [4] and the $\omega^\perp$ diagram [2] of the doubled transverse half-velocities $2\omega^\perp$ [1], including the azimuthal angles φ_i of particle emission (Fig. 1 [1)]). Taken together, these diagrams contain all the information on the kinematics of the individual event and retain the main features of its kinematic figure [1c] in velocity space - invariant geometric image of its kinematics. With the aid of the geometric criteria [5], 27 cases were discarded as π^-C events, and the remainder were distributed among the following channels: 1) ~25% without π^0 meson, 2) ~50% with $1\pi^0$, and 3) ~25% with $\geq 2\pi^0$. Two hundred events from channels 1 and 2 were classified in accordance with the types of the kinematic figures (i.e., types of kinematics of the π^-p reaction) on the basis of the principle of "similarity" of the figures and their elements, pertaining to the particle groups separated in this case and shown by the arrows in Fig. 1. The principle covers and generalizes all the kinematic selection criteria. To choose the most probable hypothesis concerning the type of the figure, it is convenient to use the likelihood-ratio criterion [6] (see, e.g., [4]). It is possible to set in correspondence with each type of figure a set (graph) of relations [1c] for the particles taking part in the reaction (Fig. 1).

In most events, we separated two (rarely, three) groups of 1 - 4 particles with and without the nucleon and with masses (as a whole) M* and $\mathfrak{M}$ (M_p, m_π, and $\mathfrak{M}_3^*$). The mass centers (CM) of the groups (asterisks in Fig. 1) are usually farther from one another than from the primary particles, and are close to a straight line joining the latter; the CM of

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1) Coordinates: Cartesian $\xi^\perp = \arctan(p^\perp/mc)$, $\rho^\parallel = \text{arc cosh}[E\cos\xi^\perp)/mc^2]$ on the L-diagram, and polar $2|\vec{\omega}^\perp| = 2T^\perp/p^\perp C$, $\varphi = \arg(\vec{\omega}^\perp)$ on the $\omega^\perp$ diagram. The coordinate grids are given in [2,4].