creases with increasing t. The entire inhomogeneity spreads out in the course of time as a result of the nonlinear dispersion of the drift velocity. Nonlinear dissipation of the inhomogeneity is observed.

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## USE OF IMAGES OF VELOCITY SPACE TO STUDY THE KINEMATICS OF $\pi^- p$ INTERACTIONS AT 7.5 GeV WITH IDENTIFIED PROTON

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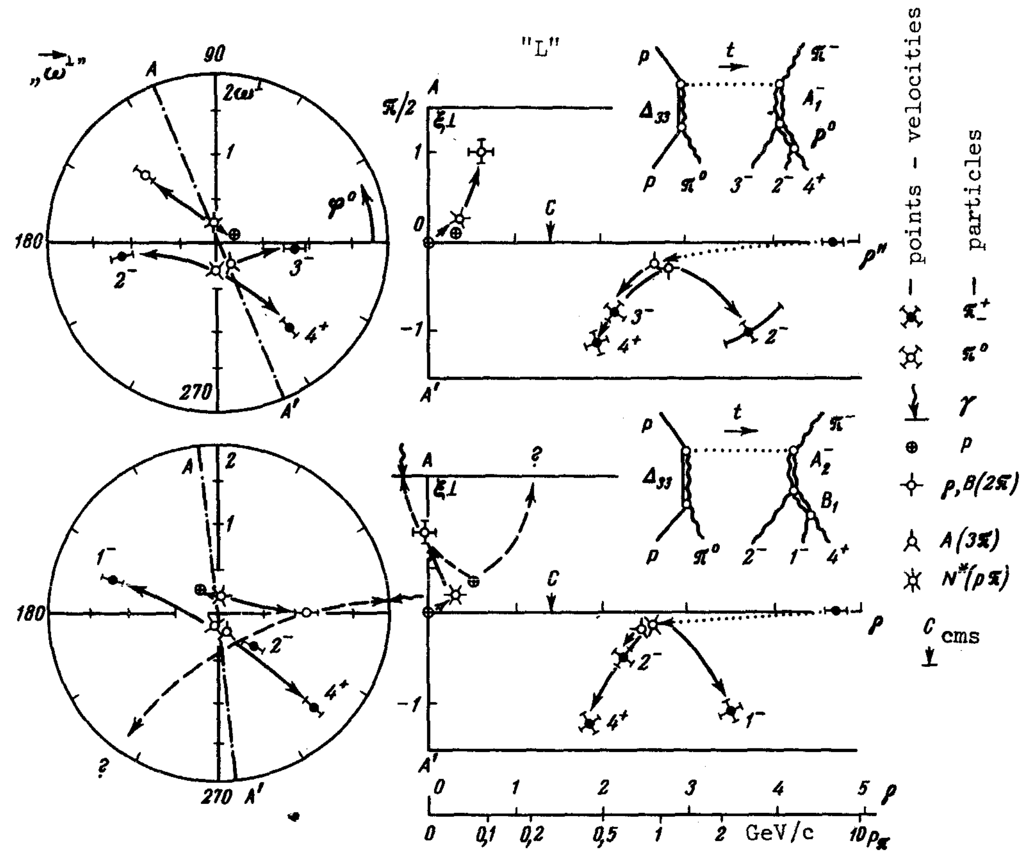
Using the new method of analyzing kinematics in images of Lobachevskii velocity space [1], proposed by one of the authors (Bubelev) [2], we analyzed anew 300 of the ~1300 4-prong $\pi^- p$ interactions in propane at energy 7.5 ± 0.6 GeV, which were investigated in [3], namely, events with the proton identified by ionization ($p_p \lesssim 1$ GeV/c) from [3a].

We used in the method the longitudinal L-diagram of Lobachevskii velocities [4] and the $\omega^\perp$ diagram [2] of the doubled transverse half-velocities $2\omega^\perp$ [1], including the azimuthal angles $\varphi_i$ of particle emission (Fig. 1 [1)]). Taken together, these diagrams contain all the information on the kinematics of the individual event and retain the main features of its kinematic figure [1c] in velocity space - invariant geometric image of its kinematics. With the aid of the geometric criteria [5], 27 cases were discarded as $\pi^- C$ events, and the remainder were distributed among the following channels: 1) ~25% without $\pi^0$ meson, 2) ~50% with $1\pi^0$, and 3) ~25% with $\geq 2\pi^0$. Two hundred events from channels 1 and 2 were classified in accordance with the types of the kinematic figures (i.e., types of kinematics of the $\pi^- p$ reaction) on the basis of the principle of "similarity" of the figures and their elements, pertaining to the particle groups separated in this case and shown by the arrows in Fig. 1. The principle covers and generalizes all the kinematic selection criteria. To choose the most probable hypothesis concerning the type of the figure, it is convenient to use the likelihood-ratio criterion [6] (see, e.g., [4]). It is possible to set in correspondence with each type of figure a set (graph) of relations [1c] for the particles taking part in the reaction (Fig. 1).

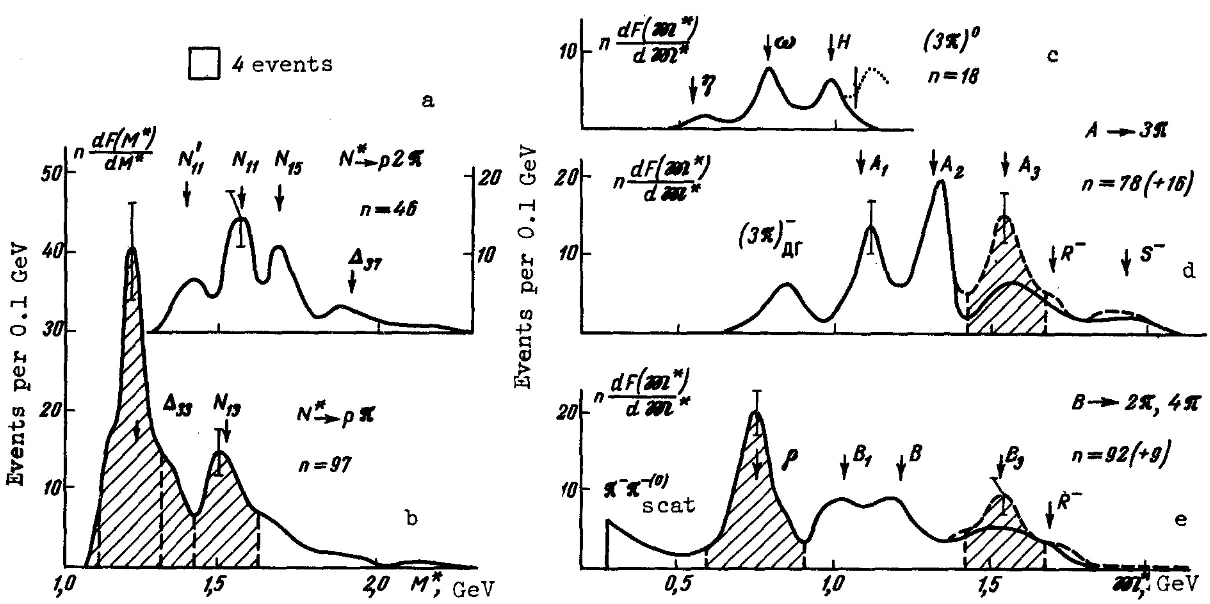
In most events, we separated two (rarely, three) groups of 1 - 4 particles with and without the nucleon and with masses (as a whole) M* and $\mathfrak{M}$ ($M_p$, $m_\pi$, and $\mathfrak{M}_3^*$). The mass centers (CM) of the groups (asterisks in Fig. 1) are usually farther from one another than from the primary particles, and are close to a straight line joining the latter; the CM of

1) Coordinates: Cartesian $\xi^\perp = \arctan(p^\perp/mc)$, $\rho^\parallel = \text{arc cosh}[E\cos\xi^\perp)/mc^2]$ on the L-diagram, and polar $2|\vec{\omega}^\perp| = 2T^\perp/p^\perp C$, $\varphi = \arg(\vec{\omega}^\perp)$ on the $\omega^\perp$ diagram. The coordinate grids are given in [2,4].



"L"
90
180
270
$\Delta_{33}$
$A_1^-$
$A_2^-$
$B_1$
points - velocities
particles
$p, B(2\pi)$
$A(3\pi)$
$N^*(p\pi)$
C cms
GeV/c


Fig. 1



4 events
a
b
c
d
e
$N^* \to p2\pi$
$n=46$
$N^* \to p\pi$
$n=97$
$\Delta_{33}$
$N_{11}$
$N_{15}$
$\Delta_{37}$
$N_{13}$
Events per 0.1 GeV
$M^*$, GeV
$(3\pi)^0$
$n=18$
$A \to 3\pi$
$n=78(+16)$
$A_1$
$A_2$
$A_3$
$B \to 2\pi, 4\pi$
$n=92(+9)$
$B_1$
$B_3$
scat
GeV


Fig. 2

the third group $\mathcal{M}_3^*$ is close to the point C - the cms of the reaction. The angular distributions in the CM systems of the pion groups are symmetrical with probability (on the whole) 53% in accord with the sign criterion [7]. The total kinetic energy of the groups in the cms is $\sum_{i=1}^{2(3)} T_i^* = E_c$ $(M^* + \mathcal{M}^*$ or $M_p + \mathcal{M}_3^* + m_\pi) = 0.7 + 2.1$ GeV, and amounts to an appreciable fraction of the total energy $E_c = \sum_i \epsilon_{ci} = 3.87$ GeV (this confirms the good separation of the groups in velocity space). The integral distributions relative to the square of the 4-momentum (-t) transferred to these groups have the same form

$$\ln n(> -t) = \ln (C/A) - A(-t),$$

as for the binary channels of the $\pi^+p$ reaction at 8 GeV/c [8]. For the subtypes of the $\pi^-p$ reaction, differing in the mass of the group with the proton $M_p = 0.94$, $M^*_{p\pi} \lesssim 1.3$, and $M^*_{p\pi, p2\pi} > 1.3$ GeV/c$^2$, the slopes are respectively $A = 5 \pm 1$, $2.5 \pm 0.5$, and $1.3 \pm 0.1$ (GeV/c)$^{-2}$ compared with $A = (7 - 9) \pm 1$ and $(2 - 12) \pm (1 - 2)$ for channels with $M_p = 0.94$ and $M^*$, $\mathcal{M}^* \lesssim 1.3$ GeV/c$^2$ in [8]. (The distributions for $p\pi$ and $2p\pi$ combinations which do not enter in $M^*$ and constitute the kinematic "background" have a slope $A \leq 0.4 \pm 0.01$ (GeV/c)$^{-2}$.) All this points to excitation of the colliding (and virtual) particles in "binary" $\pi^-p \to M^* + \mathcal{M}^*$ (and "ternary" $\pi^-p \to M_p + \mathcal{M}_3^* + m_\pi$) types of reaction without formation of a single excited system.

For a more complete utilization of the information extracted above, the densities of the distributions by masses $M^*(p\pi, p2\pi)$ and $\mathcal{M}^*(3\pi, 2\pi, 4\pi)$ for particle groups of the same type are represented not by histograms, but by "diffrograms" (Figs. 2, a - e). This is a natural designation for the derivatives $f(M^*) = n\, dF_I(M^*)/dM^*$ [6] of the smooth functions $nF_I(M^*)$, which approximate the experimental integral distributions $nF_n(M^*)$ of the volume n. The $nF_I$ agree with the corresponding $nF_n$, with probabilities $P_I(f) = 44$, 65, 62, 20, and 40% as a whole (and $P_I(f_i^m) \simeq 50 - 90$% by parts), and the alternatives $nF_{II}$ (with uniform density $\bar{f}$) agree with $nF_n$ with probabilities $P_I(\bar{f}) = 0.4$, 1, 4, $2 \times 10^{-4}$ and $3 \times 10^{-3}$ % in accordance with the strongest criterion $\pi_1$ [7]. The maxima $f_i^m$ of the diffrograms coincide with the known resonances [9], which are indicated by arrows and symbols. The statistical errors are equal to $\Delta f_i^m = f_i^m/\sqrt{n_i}$, where $n_i$ is the area of the "bell" under the i-th maximum (shaded in Fig. 2); the resolution is 2.5 - 4% for $M^*$ and 6 - 8% for $\mathcal{M}^*$. (The distributions for the remaining combinations of particles from different groups, and also within the groups ("background") have a volume 3 - 6 times larger and differ greatly from those shown in Fig. 2.) The distributions of the particles from the region of the largest maxima (with respect to $\cos\theta^*$ for the isobar $\Delta_{33}$ and the $\rho$ meson, and on the Dalitz diagram for the $A_1$ and $A_2$ mesons) do not contradict their quantum numbers.

The maxima of the diffrograms $f(M^*)$ and $f(\mathcal{M}^*)$ are interpreted as excitation levels of the nucleon and meson, respectively. These levels combine in more than 20 binary and 2 - 3 ternary physical $\pi^-p$ reaction channels of three types. The binary channels are: I (~82%) - with resonance excitation of one or both colliding particles; II (~13.5%) - with excitation of the nucleon to the levels $\Delta_{33}^{++(+)}$, $N^+(1.4)$, and $\pi^-\pi^{-(0)}$ - with scattering, and also with diffraction generation $(3\pi)^-_{DG}$ without proton excitation (Figs. 2d, e); III (~4.5%)

- ternary channels with excitation of virtual meson (vacuum), transferring the interaction between the colliding particles, to the levels of $\rho$ and $A_1$, and sometime of one of the colliding particles.

The maxima in the regions of $\mathcal{M}^*(3\pi)$, $\mathcal{M}^*(2\pi, 4\pi) = 1.4 - 1.7$ GeV/c$^2$ of the diffrograms 2d, e, supplemented with the values $\mathcal{M}^* > 1.35$ GeV/c$^2$ from the 4-prong events with the slow neutron ($P_n \leq 1$ GeV/c) (dashed), point to the existence of the new resonances, designated $A_3$ and $B_3$ mesons, observed in [10] ($P_{I,II}(A_3) = 61$ and 1.3%; $P_{I,II}(B_3) = 83$ and 11%). Their masses and widths, with the errors, were estimated with the aid of the median and sextiles [6] and their 16% confidence limits: $\mathcal{M}^*(A_3) = 1.55 \pm 0.02$, $\Gamma < 0.16 \pm 0.035$ GeV/c$^2$, and $\mathcal{M}^*(B_3) = 1.54 \pm 0.024$, $\Gamma < 0.17 \pm 0.05$ GeV/c$^2$. The isotopic spins of $A_3$ and $B_3$ are equal to $I \geq 1$. An indication was obtained that in the case of the $A_3$ meson the prevailing decay modes are $A_3 \to \pi + (B_1(1.04)$ or $B_2(1.34))$ [2)]. A new mode of $A_2$-meson decay was observed:

$$A_2 \to \pi + B_1(1.04) \to 3\pi.$$

The employed method is applicable in the region $E_{lab} > 5$ GeV under the condition $\bar{\epsilon}^* < \bar{\epsilon}_c$ for medium energies of the decay products of the sought quasiparticles (resonances): $\bar{\epsilon}^*$ in the rest system of the latter and $\bar{\epsilon}_c$ in the cms of the reaction. This method separates more distinctly the physically important information on the reaction kinematics. Owing to the reduced role of the unseparated background in the distributions 2a - d, to a 30 - 50% level, we obtained three statistically significant maxima ($0.2 < P_{II}(f) \leq 1.3\%$), and six (or seven) less significant ones ($4 < P_{II}(f) < 12\%$) compared with one $\Delta_{33}^{++}$ in [3]. For a detailed study of the revealed singularities of the $\pi^-p$ reactions, more extensive material is needed.

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2) $B_{1,2}$ mesons with masses 1.04 and 1.34 GeV/c$^2$ were observed via the 4-pion decay mode in [11] - $B_1(1.04)$ and [12] - $B_2(1.34)$.