ANOMALOUS SCATTERING OF SLOW NEUTRONS AND ELECTROMAGNETIC WAVES IN FERROMAGNETS WITH SMALL MAGNETIC ANISOTROPY

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As is well known, anomalously intense (critical) fluctuations arise in condensed media near the phase-transition point and lead to anomalously intense scattering of light (critical opalescence) and of slow neutrons in such media. We shall show that these phenomena should be observed in magnetically-ordered crystals near the point at which the magnetic symmetry of the crystal changes[1].

At small values of the effective magnetic anisotropy $\beta^* = \beta + H_0/M_0$ (β is the magnetic anisotropy constant, $\vec{M}_0$ is the equilibrium value of the magnetic-moment density in the external field $\vec{H}_0$, $\vec{M}_0 \parallel \vec{H}_0$), an important role is assumed by the coupling between the magnetic-moment oscillations and the elastic waves. The fluctuations of the quantities characterizing the crystal can be determined, with allowance for this coupling, by following [2]. We then obtain for the correlator of the magnetic-moment density

$$\langle M_i M_k \rangle_{q,\omega} = \langle M_i M_k \rangle^{(0)}_{q,\omega} + \pi/2\, TA(\chi)\,\theta_{ik}(\omega,\chi) \sum_{\omega_i = \pm\omega_{1,2}} \delta(\omega + \omega_i); \qquad (1)$$

where the tensor $\langle M_i M_k \rangle^0_{q,\omega}$ coincides with the well-known expression for the correlator of the fluctuations of the magnetic moment on a spin wave (see [3])

$$A(\chi) = [aq^2 + \delta + \beta^* \sin^2\chi]^{-1} \qquad (2)$$

$$\omega_1 = sq\zeta^{-1/2} A^{-1/2}(\chi), \quad \omega_2 = \begin{cases} \omega_1 (\chi \ll \zeta^{1/2}) \\ sq (\chi \gg \zeta^{1/2}) \end{cases} \qquad (2a)$$

χ is the angle between $\vec{q}$ and $\vec{M}_0$, $\delta = \beta^* - \zeta(1 - \beta f^{-1})^2$; $\zeta = (f - \beta)^2 M_0^2 \rho_0^{-1} s^{-2}$ is a small parameter characterizing the magnetoelastic coupling ($\zeta \sim 10^{-4} - 10^{-6}$); ρ_0 and T are the density and temperature of the crystal, s the velocity of the transverse sound, f the magnetostriction constant, and a the exchange-interaction constant; θ_{ik} is a hermitian tensor (when $\chi \ll \zeta^{1/2}$ we have $\theta_{11}(\omega_{1,2}) = \theta_{22}(\omega_{1,2}) = 1$, $\theta_{1,2}(\omega_1) = -\theta_{12}(\omega_2) = i \operatorname{sign} \omega$; when $\chi \ll \zeta^{1/2}$ all the components $\theta_{ik}(\omega_2)$, and also the components $\theta_{ik}(\omega_1)$, with the exception of $\theta_{22}(\omega_1)$, are negligibly small, and $\theta_{22}(\omega_1) = 2$; the axis 3 is chosen along $\vec{M}_0$, and the axis 2 is perpendicular to the $(\vec{M}_0, \vec{q})$ plane. The coefficient of the δ-function in the second term of (1), which describes the fluctuations on the magnetostatic waves, increases sharply when $\delta \to 0$, becoming infinite when q = 0 (critical fluctuations). Besides the magnetic-moment fluctuations, the magnetic-induction fluctuations increase sharply near the critical points, as well as the fluctuations of the transverse components of the shear vector (but not the crystal-density fluctuations).

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[1] There are crystals (e.g., magnetite [1]) whose magnetic symmetry changes at a definite temperature. In addition, the magnetic symmetry can be changed in crystals with cubic symmetry (as well as in crystals with anisotropy of the easy-plane type) by gradually decreasing the external magnetic field, which is directed at an angle to the easiest magnetization axis.

The main mechanism of slow-neutron scattering near the critical point is the interaction of the magnetic moment of the neutron with the fluctuations of the magnetic induction (magnetic scattering). Substituting (1) in the well-known formula for the differential magnetic-scattering cross section (see [3]) we get:

$$d\sigma = d\sigma_0 + \frac{2\mu_n^2}{\hbar^5 v n_0} TA(\chi)\xi(\chi) \sum_{\omega_l = \pm\omega_1} \delta(\omega + \omega_l)\, d\mathbf{p}', \tag{3}$$

where $d\sigma_0$ coincides with the well known expression for the cross section for scattering of slow neutrons by spin waves; $\hbar\omega = (2m)^{-1}(p^2 - p'^2)$, $\hbar\vec{q} = \vec{p} - \vec{p}'$; $\vec{p}$ ($\vec{p}'$) is the momentum of the incident (scattered) neutrons, μ_n the magnetic moment and m the mass of the neutron, v its initial velocity, and n_0 the number of atoms of the ferromagnet per unit volume (the neutrons are assumed, for simplicity, to be unpolarized). The function $\xi(\chi)$ is on the order of unity ($\xi(\chi) = 2$ when $\chi \ll \zeta^{1/2}$ and $\xi(\chi) = 2$ when $\chi \gg \zeta^{1/2}$).

We see that the magnetic-scattering cross section reveals near the critical point (besides the well known maximum at $\omega^2 = \omega_s^2$, where ω_s is the spin-wave frequency) an additional sharp maximum at $\omega^2 = \omega_1^2$, connected with excitation (or absorption) of magnetoelastic waves by the neutrons. The cross section of this process per unit solid-angle interval is

$$\frac{d\sigma}{dO'} = \frac{4\mu_n^2}{\hbar^4 n_0} TA(\chi)\,\xi(\chi). \tag{4}$$

According to (4), when $\delta \to 0$ the small-angle scattering cross section increases sharply in the case of particles moving almost perpendicular to the direction of $\vec{M}_0$.

We note that if the incident neutrons are polarized along $\vec{M}_0$, then it can be shown that neutron scattering by the critical fluctuations is always accompanied by a change in their spin orientation.

Relations similar to (3) and (4) determine the electromagnetic wave scattering cross section in a ferromagnet near the critical point. In particular, in the case of low-frequency waves ($\omega_0 < \zeta^{-1/2} gM_0$), when the main scattering mechanism is interaction between the waves and the magnetic-moment fluctuations (see [3]) the cross section for scattering by magnetoelastic waves is (for $\chi \ll \zeta^{1/2}$)

$$\frac{d\Sigma}{dO'} = \left(\frac{gk_0}{c}\right)^2 \epsilon\, TA(0), \tag{5}$$

where $\vec{q} = \vec{k}_0 - \vec{k}'$; $\vec{k}_0(\vec{k}')$ is the wave vector of the incident (scattered) wave, g is the gyromagnetic ratio, and ϵ is the dielectric constant of the crystal. According to (5), $d\Sigma/dO'$ increases sharply when $\delta \to 0$ (critical opalescence). Such an increase of the cross section takes place for waves incident practically perpendicular to the direction $\vec{M}_0$, and only in the case of small scattering angles. The increase in the electromagnetic-wave and neutron scattering cross sections is connected with the fact that the crystal spin system becomes unstable at the critical point.

In conclusion, let us dwell briefly on the conditions for the experimental observation of anomalous scattering of slow neutrons and electromagnetic waves in a ferromagnet near the critical point. We note in this connection that when δ decreases the differential small-angle scattering cross sections (scattering angles $\nu \ll 1$) increase in order of magntiude by Q. times, where Q is the smallest of the five quantities

$$\left\{\frac{\zeta}{\delta},\ \nu^{-2},\ \left|\frac{\pi}{2}-\theta\right|^{-2},\ \lambda^2\zeta a^{-1}\nu^{-2},\ \lambda^2\zeta a^{-1}\left|\frac{\pi}{2}-\theta\right|^{-2}\right\}$$

(θ - angle between $\vec{p}(\vec{k}_0)$ and $\vec{M}_0$, λ - wavelength of the incident particles or waves). If, for example, $\delta \sim 0.1\zeta$, then the differential scattering cross sections can increase by one order of magnitude. In the case of electromagnetic waves of frequency $\omega \lesssim 10^{13}$ sec^{-1}, such an increase of the cross section will be observed in region of scattering angles ν (and also angles $|\pi/2 - \theta|$) up to several degrees. In the case of slow neutrons with $V \sim 100$ m/sec, in order to observe an increase of one order of magnitude in the cross section at $\delta \sim 0.1\zeta$ the required scattering angles (and also the angles $|\pi/2 - \theta|$) do not exceed 0.1°.

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[3] A. I. Akhiezer, V. G. Bar'yakhtar, and S. V. Peletminskii, Spinovye volny (Spin Waves), Nauka, 1967, p. 237.

INTERNAL BREMSSTRAHLUNG AND IDENTIFICATION OF THE MECHANISM OF BINARY DIRECT REACTIONS

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It is shown in [1, 2] (see also [3]) that the moving singularities of the amplitudes of three-particle direct reactions can be used for experimental identification of the mechanism of the process. This possibility does not exist in the case of binary reactions, so that the establishment of their mechanism is a more complicated problem.

The purpose of this paper is to call attention to a method of establishing the reaction mechanism from the extrema in the energy spectrum of the internal bremsstrahlung γ quanta. The gist of the method is that the phase difference of interfering electromagnetic waves emitted by the initial and final particles depends on the duration of the reaction, and consequently also on its mechanism [4]. The latter is determined in turn by the location and the character of the singular points of the amplitude of the process. The connection between the location of the singular points and the duration of the reaction is manifest in the fact that as the kinematic variables approach the singular point the reaction duration increases. For the indicated reasons, irregularities should be observed in the energy spectrum of the internal bremsstrahlung photons; the positions of the irregularities are determined by the locations of the singularities of the reaction amplitude, and the behavior of the spectral curve near these points is determined by the character of the singularities.

We consider the mechanism of the binary reaction