In conclusion, let us dwell briefly on the conditions for the experimental observation of anomalous scattering of slow neutrons and electromagnetic waves in a ferromagnet near the critical point. We note in this connection that when δ decreases the differential small-angle scattering cross sections (scattering angles $\nu \ll 1$) increase in order of magntiude by Q times, where Q is the smallest of the five quantities

$$\left\{\frac{\zeta}{\delta},\ \nu^{-2},\ \left|\frac{\pi}{2}-\theta\right|^{-2},\ \lambda^2\zeta a^{-1}\nu^{-2},\ \lambda^2\zeta a^{-1}\left|\frac{\pi}{2}-\theta\right|^{-2}\right\}$$

(θ - angle between $\vec{p}(\vec{k}_0)$ and $\vec{M}_0$, λ - wavelength of the incident particles or waves). If, for example, $\delta \sim 0.1\zeta$, then the differential scattering cross sections can increase by one order of magnitude. In the case of electromagnetic waves of frequency $\omega \lesssim 10^{13}$ sec^{-1}, such an increase of the cross section will be observed in region of scattering angles ν (and also angles $|\pi/2 - \theta|$) up to several degrees. In the case of slow neutrons with $V \sim 100$ m/sec, in order to observe an increase of one order of magnitude in the cross section at $\delta \sim 0.1\zeta$ the required scattering angles (and also the angles $|\pi/2 - \theta|$) do not exceed 0.1°.

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INTERNAL BREMSSTRAHLUNG AND IDENTIFICATION OF THE MECHANISM OF BINARY DIRECT REACTIONS

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It is shown in [1, 2] (see also [3]) that the moving singularities of the amplitudes of three-particle direct reactions can be used for experimental identification of the mechanism of the process. This possibility does not exist in the case of binary reactions, so that the establishment of their mechanism is a more complicated problem.

The purpose of this paper is to call attention to a method of establishing the reaction mechanism from the extrema in the energy spectrum of the internal bremsstrahlung γ quanta. The gist of the method is that the phase difference of interfering electromagnetic waves emitted by the initial and final particles depends on the duration of the reaction, and consequently also on its mechanism [4]. The latter is determined in turn by the location and the character of the singular points of the amplitude of the process. The connection between the location of the singular points and the duration of the reaction is manifest in the fact that as the kinematic variables approach the singular point the reaction duration increases. For the indicated reasons, irregularities should be observed in the energy spectrum of the internal bremsstrahlung photons; the positions of the irregularities are determined by the locations of the singularities of the reaction amplitude, and the behavior of the spectral curve near these points is determined by the character of the singularities.

We consider the mechanism of the binary reaction

$$A + x \to B + \gamma, \tag{1}$$

corresponding to a triangular diagram (Fig. 1).

Corresponding to the amplitude of the internal bremsstrahlung process

$$A + x \to B + \gamma + \gamma \tag{2}$$

is the sum of diagrams of the type of Figs. 2a and b, describing the emission of a quantum by the initial, final, and virtual particles. If the amplitudes of the virtual reactions making

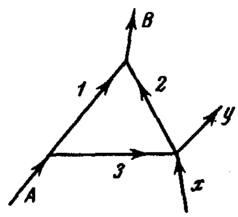

Fig. 1. Triangular diagram of binary reaction $A + x \to B + \gamma$

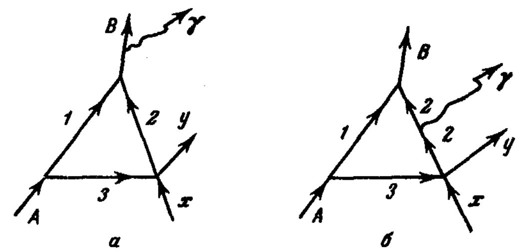

Fig. 2. Two types of diagrams of internal bremsstrahlung process

up the mechanism of the process (1) depend on the momenta, then it is necessary to take into account also photon emission from the vertices of the diagram of Fig. 1. We assume, however, that the irregularities in the spectrum of the bremsstrahlung photons from this source do not coincide with the extrema of the spectrum corresponding to the diagrams of Figs. 2a and b. The contribution made to the amplitude of the process (2) by photon emission from the vertices can then be approximate near these extrema by a certain, generally complex, constant.

The amplitude of the diagram 1 has two singularities in the variable $\omega = E_B - P_B^2/2m_B$, where P_B and E_B are the momentum and kinetic energy of the nucleus B, and m_B is its mass (if B is on the mass shell, as is the case in reaction (1), then $\omega = 0$). One of these singularities (root) corresponds to the normal threshold:

$$\omega_0 = \varepsilon_B, \tag{3}$$

where $\varepsilon_B = m_1 + m_2 - m_B$ (m_1 and m_2 are the masses of the virtual particles 1 and 2; here and throughout $h = c = 1$). The second singularity is logarithmic. Its position (ω_L) depends on the momentum transfer, or more accurately on the variable

$$t = 2(m_x - m_y)(E_x - E_y) - (\vec{P}_x - \vec{P}_y)^2, \tag{4}$$

where $\vec{P}$, E, and m are the momenta, the kinetic energies, and the masses of the particles x and y. With this,

$$\mathrm{Re}\,\omega_L = \epsilon \frac{m_B}{m_A} \frac{m_3}{m_2} (\lambda - \xi + 1). \tag{5}$$

Here

$$\varepsilon = m_1 + m_2 - M_A \tag{6}$$

$$\lambda = -\frac{m_1}{m_3}\,\frac{2(m_x - m_y)Q + t}{2m_B\,\epsilon}, \tag{7}$$

$$\xi = -\frac{m_2}{m_3}\,\frac{m_A}{m_B}\,\frac{\epsilon_B}{\epsilon}, \tag{8}$$

$$Q = m_A + m_x - m_B - m_y; \tag{9}$$

We now turn to the process of internal bremsstrahlung (2). As already mentioned, the calculation shows that the spectrum of the bremsstrahlung photons has irregularities at the points

$$\omega_\gamma = \omega_0 \text{ and } \omega_\gamma = \mathrm{Re}\,\omega_L \tag{10}$$

(ω_γ is the photon energy). Near ω_0, the differential cross section $(d\sigma/dt)_\gamma$ of the process (2) is given by

$$\left(\frac{d\sigma}{dt}\right)_\gamma = \left(\frac{d\sigma}{dt}\right)\frac{d\omega_\gamma}{\omega_\gamma}\{a + b\sqrt{|\omega_\gamma - \epsilon_B|}\}, \tag{11}$$

where $(d\sigma/dt)$ is the differential cross section of the reaction (1),

$$b = \frac{1}{2}b_0[(1+\eta)\cos\phi + (1-\eta)\sin\phi],\ \eta = \frac{(\omega_\gamma - \epsilon_B)}{(\omega_\gamma - \epsilon_B)} \tag{12}$$

and a, b_0, and ϕ are functions of the variable t. Owing to the complexity of the formulas, we do not present here the explicit expressions for these functions. At the point ω_0, the spectrum of the bremsstrahlung photons has a cusp at which the intensity reaches a maximum or a minimum, or else experiences a jump, depending on the quadrant in which the value of the phase ϕ lies. This cusp appears all the more distinctly, the smaller the value of λ (see formula (7)). Experimental observation of the cusp makes it possible to measure ϵ_B and to establish by the same token the masses of the virtual particles.

The width of the extremum at the point ω_L is determined by the value of ϵ. The general character of the spectral curve in the vicinity of ω_L is analogous to the behavior of the three-particle reaction, which is described in [2], and is determined by an amplitude term in the form

$$f_\Delta(\alpha, \xi, \lambda) = \frac{i}{\sqrt{-\lambda}}\ln\frac{\sqrt{\alpha} + \sqrt{-\xi} + \sqrt{-\lambda}}{\sqrt{\alpha} + \sqrt{-\xi} - \sqrt{-\lambda}}. \tag{13}$$

For the three-particle reactions considered in [2], we have $\alpha = 1$. The bremsstrahlung cross section contains in addition the values of f_Δ at $\alpha = \xi + \omega_\gamma m_A m_2/(\epsilon m_B m_3)$.

The extremum at the point ω_L, corresponding to the logarithmic singularity, will move, in accord with (5), when the variable t changes. The presence of such a moving extremum in the bremsstrahlung spectrum can serve as a good distinguishing attribute of the mechanism of reaction (1).

A detailed exposition of the foregoing results will be published in a more detailed article (see also [5]).

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ERRRATA

Article by A. V. Varfolomeev et al., V. 8, No. 3, p. 74:

The line above formula (1) should read: "The corresponding exciton series can be described by the formula"