It is assumed here that all the operators except ϵ_0 depend weakly on $\vec{k}$, otherwise expression (3) assumes a somewhat more complicated form, but the results remain essentially unchanged. Using (3), we obtain an expression for the differential cross section of scattering in a unit solid angle, $d\sigma/do$, in the simplified form

$$\frac{d\sigma}{do} = 5\pi^2 \frac{\omega^4}{C^4} \frac{\hbar^4 \omega_{qi}}{m^4 v_F^5} \left(\frac{\omega}{\omega_e} \frac{W}{\hbar \omega_e} \frac{e^2 a^2}{\hbar \omega_e} n \right)^2, \quad (4)$$

where n - electron density, ω_e - characteristic frequency of interband transition, and a - characteristic value of Ω_{nt}, for which we can use in estimates its classical value $a \sim \epsilon_F^{1/2} m^{-1/2} \omega_e^{-1}$.

Thus, for K with $a \sim 10^{-8}$ cm, $\omega_e \sim 10^{-16}$ sec^{-1}, $n \sim 10^{23}$ cm^{-1}, $\omega \simeq 8 \times 10^{15}$ sec^{-1} ($\omega = 4\omega_{Nd}$), $B_0 \sim 5 \times 10^4$ Oe, $v_F \sim 10^8$ cm/sec, and $W/\hbar\omega_e \sim 10^{-2}$ [5] we get $d\sigma/do \sim 10^{-10}$ cm^2. For comparison we indicate that the cross section $d\sigma/do$ for Raman scattering of ruby-laser light by nitrobenzene [6] is 5×10^{-9} cm^2.

It follows from the momentum conservation law that the wave vector $\vec{q}$ of the spin wave from which the scattering takes place satisfies the relation $g = 2k \sin(\theta/2)$, where k is the wave vector of the electron-magnetic wave and θ is the angle between the wave vectors of the incident and scattered light. Thus, by varying the observation angle θ it is possible to investigate the dispersion law of the spin waves ω_{qi} in a sufficiently wide range, including when $qR \gg 1$, where R is the cyclotron radius. For example, when $\theta \approx \pi/2$, $k \sim 10^5$ cm^{-1}, $B_0 \sim 5 \times 10^4$ Oe we have $qR \sim 50$. In an experiment aimed at observing spin waves [2] they used the phenomenon of selective transparency of metal films at the spin-wave frequency, and $qR \lesssim 1$ at reasonable values of B_0 and of the film thickness. The region of spin-wave wave vectors $qR \gg 1$ was not investigated experimentally.

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RADIATION OF SOUND BY A DISLOCATION EMERGING TO THE SURFACE OF A CRYSTAL

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We call attention in this paper to one mechanism of sound radiation by a moving dislocation, which, insofar as the author knows, has not been discussed in the literature, namely, the radiation produced when a dislocation passes through an elastic-modulus discontinuity plane, for example through grain boundaries in a polycrystal, on emerging to the surface of a crystal, etc. At the instant of passage through the boundary between two media, a sharp realignment takes place of the elastic field of the dislocation, as a result of which the field is detached, as it were, from the dislocation and propagates in the crystal in the form of a sound pulse. This radiation is analogous to the transition radiation of electromagnetic waves by a charged particle passing through a boundary between two media having different dielectric constants [1]. In analogy with electrodynamics, we shall also call the sound-radiation mechanism discussed here transition radiation.

Since it is our purpose to obtain in this paper mainly a qualitative idea of the magnitude and character of the radiation, we shall analyze it in the simplest case. Assume that a screw dislocation moves in an elastic isotropic medium filling a half-space in a direction towards the surface of the medium at a velocity V perpendicular to the surface and assumed small compared with the speed of sound. We use in the calculation a Cartesian coordinate system with the z axis along the line of emergence of the dislocation to the surface and the x axis directed along the velocity V into the interior of the medium. It follows from the symmetry of the problem that only the shear components σ_{xz} and σ_{yz} of the stress tensor and the component v_z of the velocity of the elements of the media differ from zero in the elastic field produced by the dislocation. In addition, all these quantities are homogeneous with respect to the coordinate z and depend only on x, y, and the time t. If we put $\sigma_{xx} = \sigma_1$, $\sigma_{yz} = \sigma_2$, and $v_z = v$, and let $t = 0$ be the instant of emergence of the dislocation to the surface, then the system of equations defining the elastic field of the dislocation [2] takes in this case the form

$$\begin{gathered}\frac{\partial}{\partial x}\sigma_1 + \frac{\partial}{\partial y}\sigma_2 - \rho\frac{\partial}{\partial t}v = 0,\\ \rho s^2\frac{\partial}{\partial x}v - \frac{\partial}{\partial t}\sigma_1 = 0,\\ \rho s^2\frac{\partial}{\partial y}v - \frac{\partial}{\partial t}\sigma_2 = \rho s^2 j, \quad j = -bV\delta(y)\delta(x+Vt),\end{gathered} \tag{1}$$

where b - magnitude of the dislocation Burgers vector, ρ - density of the medium, and s - velocity of the transverse sound wave. Those solutions of the system (1) which are of interest to us should have, as $r = \sqrt{x^2 + y^2} \to \infty$, the form of diverging waves of bounded amplitude, and should satisfy the condition that there are no forces on the surface of the crystal

$$\sigma_1(x, y, t)\big|_{x=0} = 0. \tag{2}$$

Denoting arbitrarily by $f(x, y, t)$ any of the quantities σ_1, σ_2, or v, we seek the solution of the system (1) in the form

$$f = f^c + f^u.$$

Here f^c is the elastic field of the dislocation in the unbounded medium, and f^u is the particular solution of the homogeneous system of equations ($j = 0$) corresponding to (1), which must be added to f^c in order to satisfy the boundary condition (2). It will be shown later that the transition radiation of interest to us is determined by the asymptotic values of the functions σ_1^u, σ_2^u, and v^u at large distances from the point of emergence of the dislocation on the surface.

The functions $f^c(x, y, t)$ and $f^u(x, y, t)$ are best represented by means of the double Fourier integrals

$$f^c(x, y, t) = \frac{1}{(2\pi)^2} \int d\omega \int dq \tilde{f}^c(\omega, q) \exp\{i[qy - \omega(\frac{x}{V} + t)]\}, \tag{3}$$

$$f^Y(x, y, t) = \frac{1}{(2\pi)^2} \int d\omega \int dq \tilde{f}^u(\omega, q) \exp\{i[qy + k(\omega, q)x - \omega t]\}, \tag{4}$$

where the symbol $\tilde{f}$ denotes the Fourier transform of the function f. The values of $\tilde{f}^c(\omega, q)$ can be obtained by substituting the system (1) in (3). In particular, for $\tilde{\sigma}_1^c(\omega, q)$ we obtain, neglecting in accord with the initial assumption the ratio v^2/s^2 compared with unity,

$$\tilde{\sigma}_1^c(\omega, q) = \frac{i\rho b s^2}{V} \frac{q}{q^2 + \frac{\omega^2}{V^2}}. \tag{5}$$

It is easy to see that the expressions for the radiation field, in the form (4), satisfy the system (1) when j = 0 and ensure satisfaction of the boundary condition (2), if we put

$$k^2 = \frac{\omega^2}{s^2} - q^2; \quad \tilde{\sigma}_1^u = -\tilde{\sigma}_1^c; \quad \tilde{\sigma}_2^u = -\frac{q}{k}\tilde{\sigma}_1^c; \quad v^u = \frac{\omega}{\rho s^2 k}\tilde{\sigma}_1^c. \tag{6}$$

To determine $k(\omega, q)$ uniquely it is necessary to assume that there is weak attenuation in the crystal. Then $k^2 = (\omega^2/s^2)[1 - i\epsilon(\omega)] - q^2$, with $\epsilon(-\omega) = -\epsilon(\omega)$ and $\epsilon(\omega) < 0$ when $\omega > 0$. The condition that the fields be bounded as $x \to \infty$ is satisfied by taking $k = k_0 \exp[i\varphi/2]$, where $k_0^2(\omega, q)$ and $\varphi(\omega, q)$ are respectively the modulus and the argument of the complex quantity $k^2(\omega, q)$.

Formulas (4) - (6) allow us to obtain general expressions for the radiation fields. For example, for the velocity of the medium we have

$$v^u(x, y, t) = \frac{1}{2\pi} \int v^{u\omega}(x, y) e^{-i\omega t} d\omega,$$

$$v^{u\omega}(x, y) = \frac{ib\omega}{2\pi} \lim_{\epsilon \to 0} \int \frac{q \exp[-x k_0 \sin\frac{\phi}{2} - i\frac{\phi}{2}]}{k_0^2(q^2 + \frac{\omega^2}{V^2})} e^{irh(q)} dq,$$

where $h(q) = q \sin\alpha + k_0 \cos\varphi/2 \cos\alpha$, $\cos\alpha = x/r$, $\sin\alpha = y/r$. At large distances from the point of emergence of the dislocation on the surface (in the wave zone) the main contribution to the spectral density of the velocity field $v^{u\omega}$ is made by integration over the vicinity of the stationary-phase points, which are determined by the equation $\partial h(q)/\partial q = 0$. It can be shown that at not too small values of the angle α such points are

$$q_1 = \mathrm{sign}(\omega)\left[\frac{|\omega|}{s}\sin\alpha + a_1\right]$$

and

$$q_2 = \mathrm{sign}(\omega)\left[\frac{|\omega|}{s} + a_2\right],$$

where a_1 and a_2 are small positive quantities that vanish together with ϵ. After integrating and going to the limit as $\epsilon \to 0$ we get

$$v^{u\omega}(x, y) = \frac{bV\exp[i\,\mathrm{sign}(\omega)\pi/4]}{\sqrt{2\pi s|\omega|}}\left[\frac{\sin\alpha}{\sqrt{r}}e^{i\frac{r}{s}\omega} + \frac{\mathrm{sign}(y)}{\sqrt{3|y|}}e^{i\frac{|y|}{s}\omega}\right]. \quad (7)$$

Similar calculations lead to the following expressions for the spectral densities of the stress-tensor components

$$\sigma_1^{u\omega}(x, y) = -\rho bV\exp[i\,\mathrm{sign}(\omega)\pi/4]\cos\alpha\sin\alpha\sqrt{\frac{s}{2\pi r|\omega|}}e^{i\frac{r}{s}\omega}, \quad (8)$$

$$\sigma_2^{u\omega}(x, y) = -\rho bV\exp[i\,\mathrm{sign}(\omega)\pi/4]\sqrt{\frac{s}{2\pi|\omega|}}\left[\frac{\sin^2\alpha}{\sqrt{r}}e^{i\frac{r}{s}\omega} + \frac{1}{\sqrt{3|y|}}e^{i\frac{|y|}{s}\omega}\right]. \quad (9)$$

We see therefore that the dislocation radiation field consists of a cylindrical wave and a plane wave propagating along the y axis.

Let us determine now the form of the sound pulse radiated by the dislocation as it emerges to the crystal surface. Integrating the radiation fields with respect to frequency, we get

$$v^u(x, y, t) = \frac{bV}{\pi}\left[\sin\alpha\frac{\theta(st-r)}{\sqrt{2r(st-r)}} + \mathrm{sign}(y)\frac{\theta(st-|y|)}{\sqrt{6|y|(st-|y|)}}\right]; \quad (10)$$

$$\sigma_2^u(x, y, t) = -\frac{\rho bVs}{\pi}\left[\sin^2\alpha\frac{\theta(st-r)}{\sqrt{2r(st-r)}} + \frac{\theta(st-|y|)}{\sqrt{6|y|(st-|y|)}}\right]; \quad (11)$$

$$\sigma_1^u(x, y, t) = -\frac{\rho bVs\sin\alpha\cos\alpha}{\pi}\frac{\theta(st-r)}{\sqrt{2r(st-r)}}; \quad \theta(x) = \begin{cases}1, & x > 0\\ 0, & x < 0\end{cases}. \quad (12)$$

Integrating with respect to frequency, we assumed the integration limits to be infinite, leading to an unphysical divergence of the fields at $t = r/s$ and $t = |y|/s$. To eliminate this divergence, it is necessary to restrict the integration limits to frequencies of order V/a, where a is the lattice parameter [3]. However, the formulas obtained thereby differ strongly from (10) - (12) only for instants of time differing from r/s and $|y|/s$ by $\Delta t \sim a/V$. Without dwelling on this question in detail, we indicate only the order of magnitude of the stresses and velocities in the indicated time interval, for a sound pulse consisting, say, of cylindrical waves:

$$v^u \sim V\sqrt{\frac{aV}{rs}}\sin\alpha; \quad \sigma_1^u \sim \rho s^2\sqrt{\frac{aV^3}{rs^3}}\sin\alpha\cos\alpha; \quad \sigma_2^u \sim \rho s^2\sqrt{\frac{aV^3}{rs^3}}\sin^2\alpha.$$

Consequently, the intensity of the pulse depends quite strongly on the velocity with which the dislocation emerges to the crystal surface.

In conclusion we note that the most convenient to observe is apparently the transition radiation of a dislocation emerging to the surface of an elastic twin, which consists, as is well known, of a large number of dislocations. Experiment shows that the velocity of the twinning dislocations can become appreciable in this case [4].

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SUPERCONDUCTING ALLOYS IN A RAPIDLY ALTERNATING MAGNETIC FIELD OF LARGE AMPLITUDE

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As already noted earlier [1], the condition for the smallness of the Joule heating of a superconductor placed in an alternating field whose amplitude is comparable with the critical field can be satisfied at a temperature that is sufficiently close to the transition temperature T_c. This raises the possibility of considering nonlinear problems. The first is that of determining the frequencies characterizing the behavior of the parameter Δ. It is clear beforehand that $\Delta(t)$ will follow the field adiabatically at external-field frequencies below a certain characteristic frequency Ω_0. Therefore to determine $\Delta(t)$ when $\omega \ll \Omega_0$ it is possible, in first approximation, to use the equations of the Ginzburg-Landau theory, in which the field depends on t as a parameter. To the contrary, when $\omega \gg \Omega_0$, we can expect Δ to respond, in the main, to the time-averaged value of the square of the field, whereas the high-frequency oscillations of Δ will have a small amplitude. In this case the time-averaged value of Δ will be determined by the Ginzburg-Landau equations, which contain the mean square of the field. Another important problem is that of the behavior of the field $\vec{A}(\vec{r}, t)$ itself. Here, too, there should exist a characteristic frequency Ω_1 that separates the region of low frequencies, at which the penetration of the field into the superconductor is determined by the Meissner effect, from the high-frequency region, where the main role is played by the skin effect. In the model of a superconductor with a large content of paramagnetic impurities,