QUANTUM SPIN WAVES

P. S. Zyryanov, V. I. Okulov, and V. P. Silin
Institute of Metal Physics, Siberian Division, USSR Academy of Sciences
Substituted 5 August 1968
ZhETF Pis Red. 8, No. 9, 489 - 492 (5 November 1968)

The classical theory of spin waves (SW) in non-ferromagnetic metals predicts a large number of limitations on the possibility of their propagation; these limitations are connected with the existence of collisionless Landau damping [1, 2]. We shall show in this communication that quantization of the orbital electron motion by a strong magnetic field uncovers a possibility of existence of new branches of spin-density oscillations, or quantum spin waves (QSW).

The energy of the elementary excitations of a degenerate electronic Fermi liquid in a constant and homogeneous magnetic field B_0 z depends on the quantum numbers n, p_z, and σ, where n - a number corresponding to the motion across the magnetic field, p_z - projection of momentum on B_0, and σ = ±1 determines the value of the spin projection. The equilibrium Fermi distribution $f(E_{n,p_z,\sigma}) \equiv f^{\sigma}_{n,p_z}$ is a function of the quasiparticle energy $E_{n,p_z,\sigma}$. We are interested in magnetic-field intensities satisfying the inequalities

$$kT < \hbar\Omega_0, \quad \hbar\Omega << \zeta, \tag{1}$$

where ζ - chemical potential, T - temperature, Ω - effective cyclotron frequency, and $\hbar\Omega_0$ - difference between the electron spin levels. The latter quantity differs from the corresponding value for the free electron, owing to effects of the Fermi-liquid interaction [1]. If inequality (1) holds, then

$$E_{n,p_z,\sigma} = E_{n,p_z} - \hbar/2\Omega_0\sigma.$$

In a weak alternating magnetic field $\delta\vec{B} = \vec{b}(\vec{r})e^{-i\omega t}$ we can obtain an equation for the nonequilibrium spin density matrix

$$\delta\sigma_{\alpha\alpha'} = 2\Sigma_{\sigma,\sigma'}S_{\sigma,\sigma'}\delta\rho^{\sigma\sigma'}_{\alpha\alpha'},$$

where δ - complete set of orbital quantum numbers of the electron, S - spin operator, and $\delta\rho^{\sigma\sigma'}_{\alpha\alpha'}$ - nonequilibrium part of the quasiparticle density matrix. Assuming that $\delta\sigma_{\alpha\alpha'}$, B_0, we get

$$[\hbar(\omega-\Omega_0) + E_{\alpha'} - E_\alpha + i\nu]\delta\sigma^+_{\alpha\alpha'} - (f^{+1}_{\alpha'} - f^{-1}_{\alpha})\left[\frac{1}{2}\sum_{\alpha_1\alpha_2}\psi^{\alpha_2\alpha'}_{\alpha\alpha_1}\delta\sigma^+_{\alpha_1\alpha_2} + \mu_0 b^+_{\alpha\alpha'}\right] = 0, \tag{2}$$
$$(\nu \to +0)$$

where $\delta\sigma^+ = \delta\sigma_x + i\delta\sigma_y$, $\psi^{\alpha'\alpha_2}_{\alpha\alpha_1}$ determines the spin-dependent part of the Fermi-liquid interaction, and μ_0 - magnetic moment of the free electron. Equation (2) does not take into account the electron collisions, the influence of which is assumed to be small. To reveal the QSW of interest to us it is sufficient to study the simplest case of waves propagating along $B_0 \| z$. Such waves are described by the density matrix

$$\delta\sigma^+_{\alpha\alpha'} = \delta\sigma^+(n_\alpha, p_{z\alpha}q)\delta_{n_\alpha,n_{\alpha'}}\delta_{p_{y\alpha},y_{\alpha'}}\delta_{p_{z\alpha},p_{z\alpha'}-\hbar q_z},$$

where $q_z \equiv q$ - wave vector of SW. We shall henceforth assume a model that takes into account approximately the Fermi-liquid interaction [3] with the aid of a single constant ψ, and we put, in the case $B_0 \parallel q$ considered by us,

$$\sum_{\alpha_1\alpha_2} \psi^{\alpha_2\alpha'}_{\alpha\alpha_1} \delta\sigma^{+}_{\alpha_1\alpha_2} = \psi \sum_{\alpha_1} \delta\sigma^{+}(n_{\alpha_1}, p_{z\alpha_1}, q) . \tag{3}$$

Recognizing that the Fourier component of the magnetization is $\delta m(q) = \mu_0 \sum_{\alpha} \delta\vec{\sigma}(n, p_z, q)$ and following [1], we get from (2) the dispersion equation

$$1 = \frac{\psi}{(2\pi\hbar)^2} \frac{|e| B_0}{c} \sum_n \int dp_z \frac{f^{+1}_{n,p_z - \hbar q} - f^{-1}_{n,p_z}}{\hbar(\omega - \Omega_0) + E_{n,p_z - \hbar q} - E_{n,p_z} + i\nu} . \tag{4}$$

Here c - speed of light, e - electron charge, and $\nu \to +0$. This equation goes over into the classical equation when $\hbar\Omega, \hbar\Omega_0 << kT$ [1]. In the simplest case when $T = 0$ we get from (4)

$$1 - G(q, u) + i\pi \frac{\psi}{(2\pi\hbar)^2} \frac{|e| B_0}{c} \sum_{n=0}^{N} \frac{m^*_u}{\hbar q} \left(\int_{-v^+_n}^{v^+_n} - \int_{-v^-_n}^{v^-_n}\right) dv\, \delta(u - v) = 0, \tag{5}$$

where

$$u = (\omega - \Omega_0)/q, \quad G(q, u) = \frac{\psi |e| B_0}{(2\pi\hbar)^2 c} \sum_{n=0}^{N} \frac{m^*_\zeta}{\hbar q} \ln \left| \frac{v^+_n - u}{v^+_n + u} \, \frac{v^-_n + u}{v^-_n - u} \right| ,$$

$$m^* = \frac{\partial^2 E_{n,p_z}}{\partial p_z^2}, \quad m^*_\zeta = m^*|_{E_{n,p_z} = \zeta} \quad m^*_u = m^*|_{p_z = p_u},$$

and p_u is the root of the equation

$$u = \partial E_{n,p_z}/\partial p_z; \quad v^{\pm}_n = \left(\frac{\partial E_{n,p_z}}{\partial p_z}\right)_{p_z = p^{\pm}_n} ,$$

with p^+_n and p^-_n respectively the roots of the equations

$$\zeta + \frac{\hbar\Omega_0}{2} - E_{n,p_z - \hbar q} = 0 \text{ и } \zeta - \frac{\hbar\Omega_0}{2} - E_{n,p_z} = 0;$$

and finally N is the integer part of $\zeta/h\Omega$.

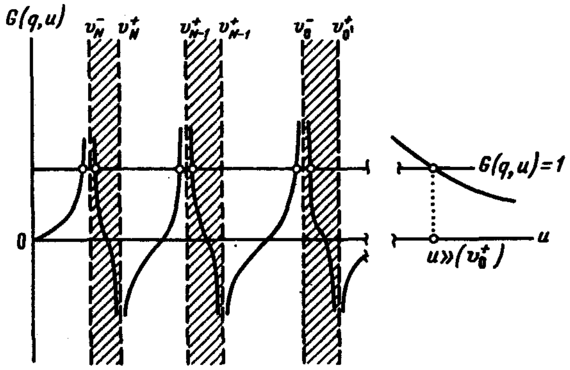
Since $v^+_n > v^-_n$, the first integral in (5) yields unity when $v^-_n < u < v^+_n$, and the second integral yields zero, so that the imaginary part is of the same order as the real one. If $v^-_{n-1} > v^+_n$ (as is always the case when $\Omega > \Omega_0$) and $v^+_n < u < v^-_n$, then the imaginary part of (5) vanishes. When $\psi > 0$ and $\Omega_0 > \Omega$ the inequality $v^+_n < v^-_{n-1}$ is impossible and the imaginary part of (5) is not small for all $u < (v^+_n)_{max} = v^+_0$, but always vanishes when $u > v^+_0$. As a net result, undamped solutions $u = u(q)$ of (5) can exist only when $v^-_{n-1} > v^+_n$, in the interval $v^+_n < u < v^-_{n-1}$, and always when $u > v^+_0$. Retaining in (5) only the real part, we obtain at small values of q the following solutions:

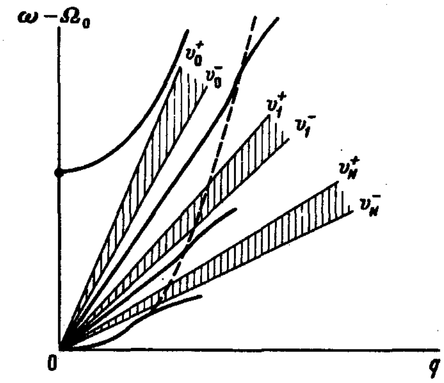
when $u >> v^+_0$,

$$\omega_1 = \Omega_0 \left[1 + \frac{\psi |e| B_0}{(2\pi\hbar)^2 c} \sum_{n=0}^{N} \frac{2m^*_\zeta}{\hbar} (v^-_n - v^+_n)\right] + 0(q^2), \tag{6}$$

when $u << (v^-_n)_{min}$,

$$\omega_2 = \Omega_0 + 0(q^2), \tag{7}$$





when $\lim_{q=0} u = \text{const}$, there exist (2N + 1) solutions of the type

$$\omega_l = \Omega_0 + c_l q, \quad (8)$$

and the values of c_i are obtained from the equation

$$\sum_{n=0}^{N} \ln \frac{v_n^+ - c_l}{v_n^+ + c_l} \cdot \frac{v_n^- + c_l}{v_n^- - c_l} = 0.$$

The solutions (6) - (8) can be obtained by plotting G(q, u) at fixed values of q and finding the points of intersection of G(q, u) and the line G = 1. Such a plot is shown in Fig. 1. The damping in the shaded regions is large, so that N + 2 out of the 2N + 3 branches of the spectrum are undamped. Undamped quantum electromagnetic waves were obtained in [4]. The thick lines in Fig. 2 show the approximate dispersion curves ω(q), and the shaded sections of the (ω, q) plane show the regions of strong damping. The spectrum (6) (upper curve of Fig. 2) corresponds to the classical limit [1], whereas the QSW (7) and (8) are possible only in the quantum state when $\Omega_0 < \Omega$.

On the other hand, if $\Omega_0 > \Omega$, then the QSW are impossible, owing to the large Landau damping. An analysis of (5) shows, however, that even if $\Omega_0 > \Omega$, but if $\psi < 0$, undamped QSW are possible if q exceeds q* and depends on ψ and $(\Omega_0 - \Omega)$.

[1] V. P. Silin, Spin Waves in Nonferromagnetic Metals. Supplement to the book by A. I. Akhiezer, V. G. Bar'yakhtar, and S. V. Peletminskii, Spinovye volny (Spin Waves), Nauka, 1967.
[2] D. R. Fredkin, J. R. Buchler, and A. R. Wilson, Bull, Am. Phys. Soc., ser. II, No. 3, 362 (1968).
[3] V. P. Silin, Zh. Eksp. Teor. Fiz. 55, 697 (1968) [Sov. Phys.-JETP 28 (1969)].
[4] E. A. Kaner and V. G. Skobov, Phys. stat. sol. 22, 333 (1967).

VIOLATION OF CP-INVARIANCE IN WEAK ELECTROMAGNETIC AND MINIWEAK PARITY CONSERVING INTERACTIONS

B. A. Arbuzov and A. T. Filippov
Joint Institute for Nuclear Research
Submitted 2 September 1968
ZhETF Pis. Red. 8, No. 9, 493 - 496 (5 November 1968)

Experimental studies of K^0-meson decays and searches for CP-violation in other processes have not explained so far the mechanism of this violation. Moreover, recent experimental data [3, 4] make it necessary to return to a discussion of the mechanisms that con-