$$\epsilon(k) = L(0) \pm \left\{\left[\frac{H}{2}\left(\frac{I_{11} + I_{12}}{H_{2c}} - g\mu_0\right) \pm |L(k)|\right]^2 + \frac{1}{4}\left(1 - \frac{H^2}{H_{2c}^2}\right)(I_{11} - I_{12})^2\right\}^{1/2},$$

3) finally, in strong fields ($H \geq H_{2c}$)

$$\epsilon(k) = L(0) \pm \left[\frac{H}{2}\left(\frac{I_{11} + I_{12}}{H_{2c}} - g\mu_0\right) \pm |L(k)|\right].$$

Being unable to discuss all the features of the spectrum, we note only that whereas in weak fields the gap does not depend essentially on the field and does so only as a result of the Zeeman effect, the dependence of the gap on the field becomes quite appreciable even in medium fields and is determined by the exchange interaction of the s electrons with the magnetic sublattices that collapse in the field H_{2c}. The gap then vanishes and the semiconductor becomes a metal. The spectrum in the third region is already characteristic of a ferromagnetic metal, and it can be readily seen that it is necessary to go over to the crystal-chemical cell. We disregarded the reaction of the conduction electron on the system of the d electrons, and also diamagnetic effects.

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EXPLANATION OF INTERFERENCE EXPERIMENTS WITH H I AND He II

D. A. Varshalovich
A. F. Ioffe Physico-technical Institute, USSR Academy of Sciences

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Very interesting results of experiments performed in the USA on interference between the levels of the fine structure of H I and He II, excited by passing through thin foils, have recently been published [1-4]. The authors were unable to explain fully the experimental results obtained by them. The intensity of some of the lines of the characteristic radiation emitted by the particles passing through the foil varied along the beam not in monotonic fashion, but oscillated. Electric or magnetic fields were applied to the beam, several foils were installed, different initial ions were used (e.g., H^+, H_2^+, H_3^+), etc. Beats of the radiation intensity of the H_β, H_γ, H_δ, and H_ϵ, i.e., the lines n = 4, 5, 6, 7 → n = 2, were observed in the case of H I, and the lines n = 7, 8, 9, 10 → n = 4 in the case of He II.

The theoretical explanation presented in [1,2] was based on the assumption that passage through the foil causes states with different energies to be excited statistically independently, and only superposition of an external field gives rise to superposition of states with different energies, leading to beats of the intensity. From our point of view this is not the case. Coherent superposition of states with different internal energy arises already after the passage through the first foil, even in the absence of an external field. The density matrix $\rho_{bb'}(x)$ characterizing the state of the particles in the beam that passed the foil at a distance x from it will not be diagonal with respect to the energy ϵ within limits $\Delta\epsilon \sim \hbar/\Delta t$ (Δt - time of passage through the target). For a thin foil

$$\rho_{bb'}(x) = \sum_a \rho_{aa}(S^+_{ba}\, S_{ab'})e^{(i\omega_{bb'} - \gamma_{bb'})\frac{x}{v}}, \tag{1}$$

where the diagonal matrix ρ_{aa} characterizes the initial state of the particles prior to passing through the foil. S_{ab} is the amplitude of the transition from the state a into the state b as a result of interaction with the foil. The parameters ω_b and γ_b characterize the average energy and the energy width of the state,

$$\omega_{bb'} = \omega_b - \omega_{b'}, \quad \gamma_{bb'} = \frac{\gamma_b + \gamma_{b'}}{2}.$$

The intensity of the radiation occurring on going from such a state bb', which is not diagonal in the energy, into a state c, is given by

$$I(x) = \rho^{(0)}_{bb}|a_{bc}|^2 e^{-\gamma_b \frac{x}{v}} + \rho^{(0)}_{b'b'}|a_{b'c}|^2 e^{-\gamma_{b'}\frac{x}{v}} + \\ + 2\,|a^+_{cb}\,\rho^{(0)}_{bb'}a_{b'c}|\, e^{-\gamma_{bb'}\frac{x}{v}}\cos(\omega_{bb'}\frac{x}{v} + \phi), \tag{2}$$

where a_{bc} - amplitude of probability of emission of a photon with definite polarization in the observation direction, and $\varphi = \arg(a_{cb}\rho^{(0)}_{bb'}a_{b'c})$ - initial phase.

Thus, the beats of the radiation intensity are due to the fact that in the intermediate state the internal energy of the system is not fixed in one and the same atom, and can be in the states b and b' with different energy at the same time. In the final state, to the contrary, the system should have a definite energy. Therefore the frequency of the beat reflects the splitting of the upper level and does not depend on the splitting of the lower level.

To produce an interference pattern it is necessary, first, that the upper level have the corresponding splitting $1/\Delta t \gtrsim \omega_{bb'} \gtrsim \gamma_{bb'}$. Second, it is necessary that these interfering sublevels be excited coherently on passing through the foil, $\langle S^+_{ba}S_{ab'}\rangle \neq 0$. Third, it is necessary that a transition to the same final state $\langle a^+_{cb}a_{b'c}\rangle \neq 0$ be possible from these coherently excited sublevels. Fourth, it is necessary that the polarizztions of the interfering radiation components not be orthogonal, or else it is necessary to use a polarimeter.

The level splitting can have an arbitrary nature and may be due not only to the interaction with the external fields, but also to internal causes, for example the Lamb splitting. However, in the case of H I, the interference of the components of the Lamb doublet $n(S_{1/2} - P_{1/2})$ or $n(P_{3/2} - D_{3/2})$ should not take place in the absence of an external field, since the levels with different parity $2(S_{1/2} - P_{1/2})$ do not overlap in the final state, and the transition from different levels of the Lamb doublet to one and the same final state is impossible.

The situation is different in the case of He II, for in the final state the levels of different parity $4(P_{3/2} - D_{3/2})$ and $4(D_{5/2} - F_{5/2})$ overlap [5]. Therefore, interference is possible in the case of He II even in the absence of an external field, as is indeed observed experimentally [4]. However, the authors were quite surprised by this result, since they assumed that beats cannot occur without an external field, and explained this result as being exclusively due to the presence of induced fields in their setup.

On moving along the beam, the interference pattern becomes smeared out, first as a result of the spread in the values of x/v, due in particular to straggling and to the finite angular resolution of the spectrograph, and second, as a result of the fact that additional population of the levels of interest to us takes place as a result of spontaneous transitions from higher levels, which are also excited on passing through the foil, the degree of coherence decreasing in this case.

Experimental and theoretical values of the frequencies f (in units of 10^{-7} sec^{-1}).

HI $n \to n$	f_{exp}, $H = 8$ G $E^* =$ =29 V/cm [1]	f_{theor} $E_{eff} =$ =36V/cm	HeII $n \to n$	f_{exp}, $E^* =$ =40V/cm [4]	f_{theor} $E_{eff} =$ 36 V/cm	f_{exp}, $E^* =$ =60V/cm [4]	f_{theor} $E_{eff} =$ = 52 V/cm
4 → 2	52 ± 13	70	7 → 4	–	48	77 ± 8	70
5 → 2	76 ± 19	56	8 → 4	56 ±6	55	82 ± 8	80
6 → 2	84 ± 8	84	9 → 4	61 ± 6	62	88 ±9	90
7 → 2	96 ± 10	98	10 → 4	–	69	96 ±10	100

The authors of [4] did not explain the frequency of the beats observed by them. Yet it is determined by the usual Stark splitting [5] and corresponds to beats between the nearest Stark components with identical polarization, i.e., with $\Delta(n_1 - n_2) = 2$

$$f \text{ (MHz)} = 3.84 \, E \text{ (V/cm)} \, n/z. \tag{3}$$

The values of f_{exp} [1,4] agree well with (3) in the sense of the dependence on the field intensity E and on the principal quantum number of the upper level n (see the table). However, to calculate the absolute value of f it is necessary to take into account the fact that the effective field in the beam E_{ef}, as noted by the authors of [4] themselves, differed from the applied external field E^*, owing to induction and the presence of space charge in the beam. We note also that the earth's magnetic field could produce $\Delta E \sim 4$ V/cm.

Of course, f_{exp} is only the effective mean value, since the interfering Stark sublevels at $E \sim 50$ V/cm are not strictly equidistant. The overall picture of the beats I(t) is determined by the large number of components with different frequencies. Taking the Fourier transform of the experimental I(t) curves, it is possible to obtain from the form of the $I(\omega)$ spectrum and from its dependence on E more detailed information on the splitting and the individual populations of the interfering sublevels, to reconstruct the density matrix $\rho_{bb'}^{(0)}$ characterizing the state of the particles directly after the interaction with the foil, and consequently to determine more accurately the character and parameters of this interaction for different ions (H^+, H_2^+, H_3^+) with different foils.

In conclusion we note that an interpretation of experiments of this type was presented by us in [6], and an interpretation of similar experiments with K mesons is given in [7,8].

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DISPERSION SUM RULES FOR THE FORM FACTOR OF THE π MESON

L. A. Khalfin and Yu. P. Shcherbin
Leningrad Division, V. A. Steklov Mathematics Institute; Leningrad State University

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1. One of the authors (L. A. Kh.) [1] obtained dispersion sum rules (DSR) for the form factors of elementary particles. These DSR connect the behavior of the form factor G(t) at $t \leq 0$ with the behavior of G(t) at $t \geq t_0$, [1)] which is determined by the cross section in the annihilation channel. As indicated in [1], it is possible to obtain from these DSR an estimate [2)] of the electromagnetic radius of elementary particles.

2. In the present paper we investigate the DSR for the form factor G(t) of the π meson. In this case $t_0 = 4m_\pi^2$ and G(t) is real when $t < t_0$. The form factor G(t) with $t \leq 0$ was investigated experimentally in many papers [3]. G(t) with $t \geq t_0$ was determined for the cross section of the annihilation

$$e^- + e^+ \rightarrow \pi^- + \pi^+, \qquad (1)$$

which was recently investigated with the aid of electron-positron colliding beams [4].

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1) The notation is the same as in [1].

2) Certain similar estimates of the radius of elementary particles were given in [2].