the fact that the effective field in the beam E_{ef}, as noted by the authors of [4] themselves, differed from the applied external field E^*, owing to induction and the presence of space charge in the beam. We note also that the earth's magnetic field could produce $\Delta E \sim 4$ V/cm.

Of course, f_{exp} is only the effective mean value, since the interfering Stark sublevels at $E \sim 50$ V/cm are not strictly equidistant. The overall picture of the beats I(t) is determined by the large number of components with different frequencies. Taking the Fourier transform of the experimental I(t) curves, it is possible to obtain from the form of the $I(\omega)$ spectrum and from its dependence on E more detailed information on the splitting and the individual populations of the interfering sublevels, to reconstruct the density matrix $\rho^{(0)}_{bb'}$ characterizing the state of the particles directly after the interaction with the foil, and consequently to determine more accurately the character and parameters of this interaction for different ions (H^+, H_2^+, H_3^+) with different foils.

In conclusion we note that an interpretation of experiments of this type was presented by us in [6], and an interpretation of similar experiments with K mesons is given in [7,8].

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DISPERSION SUM RULES FOR THE FORM FACTOR OF THE π MESON

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1. One of the authors (L. A. Kh.) [1] obtained dispersion sum rules (DSR) for the form factors of elementary particles. These DSR connect the behavior of the form factor G(t) at $t \leq 0$ with the behavior of G(t) at $t \geq t_0$, [1)] which is determined by the cross section in the annihilation channel. As indicated in [1], it is possible to obtain from these DSR an estimate [2)] of the electromagnetic radius of elementary particles.

2. In the present paper we investigate the DSR for the form factor G(t) of the π meson. In this case $t_0 = 4m_\pi^2$ and G(t) is real when $t < t_0$. The form factor G(t) with $t \leq 0$ was investigated experimentally in many papers [3]. G(t) with $t \geq t_0$ was determined for the cross section of the annihilation

$$e^- + e^+ \to \pi^- + \pi^+, \quad (1)$$

which was recently investigated with the aid of electron-positron colliding beams [4].

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1) The notation is the same as in [1].

2) Certain similar estimates of the radius of elementary particles were given in [2].

3. On the basis of the methods of [1] we obtain the following DSR:

$$i \equiv \int_{t_2}^{t_1} \frac{G(t)\ dt}{\sqrt{(t-t_2)(t_1-t)(t_0-t)}} = \int_{t_0}^{\infty} \frac{\mathrm{Re}\,G(t)\ dt}{\sqrt{(t-t_2)(t-t_1)(t-t_0)}} \equiv j \tag{2}$$

$$p \equiv \int_{t_2}^{t_1} \frac{\ln G(t)\ dt}{\sqrt{(t-t_2)(t_1-t)(t_0-t)}} = \int_{t_0}^{\infty} \frac{\ln|G(t)|\ dt}{\sqrt{(t-t_2)(t-t_1)(t-t_0)}} \equiv q. \tag{3}$$

In (2) and (3) we have $-\infty < t_2 < t_1 \leq 0$. The DSR (2) and (3) differ, in particular, in the fact that when t_1 is suitably chosen it is possible to eliminate in them the region of very small $t \in [t_1, 0]$, where G(t) is determined experimentally with low accoracy. In the derivation of (2), a definite power-law limitation is imposed on the asymptotic form of G(t). The DSR (3) are valid for all power-law asymptotic forms of G(t), but it is assumed that G(t) does not have complex zeros.

4. If it is assumed that the reaction (1) proceeds primarily via the ρ meson, i.e.,

$$e^- + e^+ \to \rho \to \pi^- + \pi^+, \tag{4}$$

then we get for $|G(t)|$ with $t \geq 4m_\pi^2$ (at any rate, in the vicinity of the resonance)

$$|G(t)|^2 = \frac{k\ m_\rho^4}{(t - m_\rho^2)^2 + m_\rho^2\Gamma_\rho^2}, \tag{5}$$

where on the basis of [4] we have $k = 0.59 \pm 0.15$, $m_\rho = 764 \pm 11$ MeV, and $\Gamma_\rho = 93 \pm 15$ MeV. Under the same assumptions we get

$$\mathrm{Re}\,G(t) = \frac{(m_\rho^2 - t)\ k^{1/2}\ m_\rho^2}{(t - m_\rho^2)^2 + m_\rho^2\Gamma_\rho^2}. \tag{6}$$

The most accurate results with respect to G(t) were obtained in [5] for the momentum-transfer interval $t \in (1 - 6)f^{-2}$. The main conclusion of [5] is that within the limit of experimental errors the pion and proton charge form factors are identical and consequently G(t) for either the proton or the pion is described by the formula

$$G(t) = [1 - (t/a)]^{-2}, \tag{7}$$

where $a = 0.62 \pm 0.16\ (\mathrm{GeV}/c)^2$ for the pion.

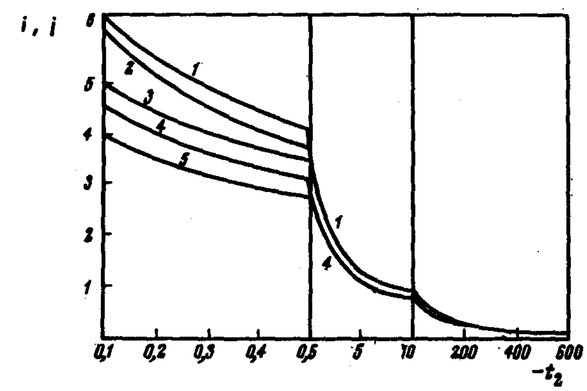

Fig. 1

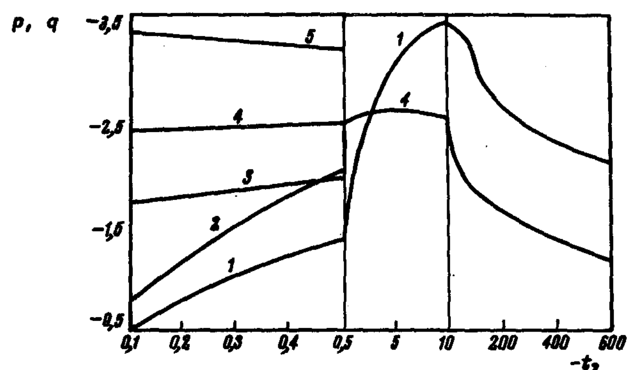

Fig. 2

Curves 1,2 - values of i and p at a = 0.72 and 0.46 $(\mathrm{GeV}/c)^2$ respectively; curves 3,4,5 - values of j and q at k = 0.74 m_ρ = 775 MeV and Γ_ρ = 108 MeV (3), k = 0.59, m_ρ = 764 MeV and Γ_ρ = 93 MeV (4), and k = 0.44 m_ρ = 753 MeV and Γ_ρ = 78 MeV (5).

5. The experimental results described in Sec. 4 were used for a numerical analysis of the DSR (2) and (3). In the system of units where $\hbar = c = 5m_\pi = 1$, we chose $t_1 = -0.02$ in (2) and (3); the values of t_2 were varied in the intervals $t_2 \in [-0.1, -0.5]$, $[-0.5, -10]$, $[-10, -600]$. We note that in our units the maximum experimentally investigated value of t corresponds to $-t = 0.48$. In the calculation we used the values of a in the interval $a \in [0.46, 0.72]$ $(\mathrm{GeV/c})^2$, where the upper limit corresponds to the nucleon $G(t)$. The values of k, m_ρ, and Γ_ρ were chosen in accordance with [4]. The calculations were performed with a computer with appropriate allowance for the integral singularities in (2) and (3). The results are shown in Figs. 1 and 2.

6. As seen from Fig. 1, at small values of t_2 there is a discrepancy between the approximation of the experimental data in accordance with (6) and (7). This can be attributed both to the fact that the local properties of $G(t)$ are poorly described by formula (6) at these small values of t, and to the fact that in the calculation of j emphasis is placed on the values of t close to t_0, for which the approximation in the form (6) is patently insufficient. With increasing $|t_2|$, the agreement between the approximation formulas (6) and (7) becomes sufficiently good, and good agreement is obtained here at values of $|t_2|$ much larger than those verified experimentally. It is possible that the agreement between i and j improves if additional account is taken in (6) of the far resonance, or else if a more accurate allowance is made for the nonresonant dependence, particularly for the threshold behavior in the vicinity of $t = t_0$. As seen from Fig. 2, the discrepancy between p and q at small values of t_2 is much larger. This is connected with the fact that in (3) emphasis is placed on those values of t (in particular, t close to t_0) at which the approximation of $G(t)$ by the ρ meson peak is clearly insufficient. With increasing $|t_2|$, the discrepancy decreases, up to $t_2 \sim -2.5$, and with further increase of $|t_2|$, it again becomes appreciable. The decrease in the discrepancies of p and q up to $t_2 \simeq -2.5$ can be attributed to the fact that with increasing $|t_2|$ in this region, the contribution of the threshold region $(t \sim t_0)$ becomes less significant and the resonant approximation (5) becomes sufficiently good. On the other hand, a deterioration with further increase of $|t_2|$ can be attributed to the fact that the extrapolation of $G(t)$ may not occur in accordance with (7), and by the fact that a more important role is assumed in (3) by values of $G(t)$ with large t, where the ρ-resonance behavior is clearly insufficient. Taking into account, however, the tendency of the curves of Fig. 1, it can be assumed that this discrepancy between p and q at large values of $|t_2|$ can be due also to the contribution of possible complex zeros of $G(t)$, which are not taken into account in (3).

7. It is easy to show that the following dispersion relation holds for $G(t)$ when $t_1 \leq t \leq t_0$:

$$\ln G(t) = \frac{[(t_0 - t)(t - t_1)(t - t_2)]^{1/2}}{\pi} \cdot \left[\int_{t_0}^{\infty} \frac{\ln|G(t')|\, dt'}{\sqrt{(t'-t_0)(t'-t_1)(t'-t_2)}(t'-t)} - \int_{t_2}^{t_1} \frac{\ln G(t')\, dt'}{\sqrt{(t_0-t')(t_1-t')(t'-t_2)}(t'-t)} \right]. \tag{8}$$

With the aid of (8) it is possible in principle to calculate the behavior of G(t) at $t \sim 0$ and by the same token estimate the electromagnetic radius of the pion. Preliminary calculations show that when $t \gtrsim t_0$ it is necessary to have a more correct representation of $|G(t)|$ than given by (5). In a recent paper [6], the radius of the pion was estimated using only the experimental data on ρ-resonance. The formula used in [6] for the estimate is a particular case of the DSR (3). In [6] they obtained a very rough estimate, since, as shown above, in the region of small t the DSR are not vergy accurate in these approximations. In principle, (8) makes use of more experimental information and one can hope to obtain a more accurate estimate for $\langle r_\pi \rangle$.

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CONCERNING ONE POSSIBILITY OF ANOMALOUSLY RAPID TURBULENT HEATING OF PLASMA IONS

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As shown in [1-4], the scattering of ions by small-scale high-frequency turbulent electron-sound pulsations is an effective mechanism of heating the ionic component of a plasma. The excitation of electron-sound oscillations can be due to the motion of the ions relative to the electrons across an external magnetic field $\vec{H}_0$ in an electric field of a low-frequency electromagnetic wave with frequency on the order of the cyclotron frequency, or in the current front of a collisionless shock wave. The time necessary to heat the ions to a final temperature $T_i \sim m_i u^2$ is determined by the energy density W of the turbulent pulsations

$$\tau \sim (1/\omega_{Hi})(n_e m_i u^2/W),$$

where u - amplitude of the relative velocity of the ions and electrons and n_e - electron density. The value of W is limited by the nonlinear interaction of the waves to a level on the order of $W \sim n_e m_e u^2$, so that the heating time $\tau \sim m_i/m_e\omega_{Hi}$ turns out to be $10^2 - 10^3$ times longer than the period of the oscillations of the low-frequency wave $\sim 2\pi/\omega_{Hi}$.

In this paper we investigate the stability of a plasma consisting of a mixture of ions of two types moving perpendicular to the magnetic field in an electric field of a low-frequency electromagnetic wave. The opposing ion currents can arise also in a collisionless shock wave [5]. The ion-ion instability resulting in this case can play an important role in the formation of the wave front and in the heating of the ions. We shall show that in a three-