With the aid of (8) it is possible in principle to calculate the behavior of G(t) at $t \sim 0$ and by the same token estimate the electromagnetic radius of the pion. Preliminary calculations show that when $t \gtrsim t_0$ it is necessary to have a more correct representation of $|G(t)|$ than given by (5). In a recent paper [6], the radius of the pion was estimated using only the experimental data on ρ-resonance. The formula used in [6] for the estimate is a particular case of the DSR (3). In [6] they obtained a very rough estimate, since, as shown above, in the region of small t the DSR are not vergy accurate in these approximations. In principle, (8) makes use of more experimental information and one can hope to obtain a more accurate estimate for $\langle r_\pi \rangle$.

[1] L. A. Khalfin, Yad. Fiz. 7, 876 (1968) [Sov. J. Nucl. Phys. 7 (1968)].
[2] Nguen Wang Hieu, ibid. 7, 1111 (1968) [7, (1968)].
[3] F. S. Crowford, Phys. Rev. 117, 1119 (1960); J. Allen et al., Nuovo Cimento 32, 1114 (1964); V. G. Grishin et al., Yad. Fiz. 2, 886 (1965) [Sov. J. Nucl. Phys. 2, 632 (1966)]; M. M. Sternheimer and R. Hofstadter, Nuovo Cimento 38, 1854 (1966); M. E. Norberg and K. F. Kinsey, Phys. Lett. 20, 692 (1966).
[4] V. L. Auslander et al., Phys. Lett. 25B, 433 (1967).
[5] C. W. Akerlof et al., Phys. Rev. 163, 5 and 1482 (1968); M. M. Block et al., ibid. 169, 1074 (1968).
[6] J. E. Bowcock and Th. Kannelopoulos, Nucl. Phys. B4, No. 4, 417 (1968).

CONCERNING ONE POSSIBILITY OF ANOMALOUSLY RAPID TURBULENT HEATING OF PLASMA IONS

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As shown in [1-4], the scattering of ions by small-scale high-frequency turbulent electron-sound pulsations is an effective mechanism of heating the ionic component of a plasma. The excitation of electron-sound oscillations can be due to the motion of the ions relative to the electrons across an external magnetic field $\vec{H}_0$ in an electric field of a low-frequency electromagnetic wave with frequency on the order of the cyclotron frequency, or in the current front of a collisionless shock wave. The time necessary to heat the ions to a final temperature $T_i \sim m_i u^2$ is determined by the energy density W of the turbulent pulsations

$$\tau \sim (1/\omega_{Hi})(n_e m_i u^2/W),$$

where u - amplitude of the relative velocity of the ions and electrons and n_e - electron density. The value of W is limited by the nonlinear interaction of the waves to a level on the order of $W \sim n_e m_e u^2$, so that the heating time $\tau \sim m_i/m_e\omega_{Hi}$ turns out to be $10^2 - 10^3$ times longer than the period of the oscillations of the low-frequency wave $\sim 2\pi/\omega_{Hi}$.

In this paper we investigate the stability of a plasma consisting of a mixture of ions of two types moving perpendicular to the magnetic field in an electric field of a low-frequency electromagnetic wave. The opposing ion currents can arise also in a collisionless shock wave [5]. The ion-ion instability resulting in this case can play an important role in the formation of the wave front and in the heating of the ions. We shall show that in a threecomponent plasma there exist, besides the well known hydrodynamic ion-ion instability [6,7], also other types of ion-ion instabilities, connected with the excitation of strongly damped modes, when the relative velocity of the ionic components is equal to zero. It is precisely these oscillations, whose phase velocity is on the order of the thermal velocity of the ions, which lead to a rapid heating of the ions. Since the nonlinear interaction of the waves limits the energy of the ion-ion oscillations to a value $W \sim n_0 m_i u^2$ m_i/m_e times larger than the value of W for the electron-sound oscillations, it follows that the heating time of the ions becomes comparable with $\tau \sim 1/\omega_{Hi}$.

The dispersion equation for the electrostatic oscillations of the plasma of frequency ω greatly exceeding ω_{Hi} but considerably lower than ω_{He}, and of wavelength considerably shorter than the ion Larmor radius but considerably longer than the electron Larmor radius, is

$$1 + \frac{\omega_{pe}^2}{\omega_{He}^2} + \sum_{\alpha = e,1,2} \frac{\omega_{p\alpha}^2}{k^2 v_\alpha^2}[1 + i\sqrt{\pi} z_\alpha w(z_\alpha)] = 0, \tag{1}$$

where

$$z_e = \frac{\omega - k u_e}{\sqrt{2} k_z v_e}, \quad z_{1,2} = \frac{\omega - k u_{1,2}}{\sqrt{2} k v_{1,2}},$$

$$\omega = \omega_k + i\gamma_k,$$

w - Kramp function,

$$\omega_{p\alpha} = (4\pi e_\alpha^2 n_\alpha / m_\alpha)^{1/2} (\omega_{p\,1,2} \gg \omega_{H\,1,2}),$$

$$v_\alpha = (T_\alpha / m_\alpha)^{1/2},$$

$\vec{u}_\alpha$ - velocities of the electrons and ions ($u_\alpha \ll v_e$), $k_z = k \cos \nu$, and ν - angle between the vectors $\vec{k}$ and $\vec{H}_0$. In the different limiting cases considered below, we shall assume for simplicity that $n_1 \sim n_2 \sim n_e$, $m_1 \sim m_2$, and $e_1 \sim e_2$.

1. If $T_e \gg T_i$, then, in addition to the known branch of oscillations of the ion-sound type [6], there may be excited one more oscillation branch existing only in a plasma with different types of ions. For these oscillations, the electronic terms in (1) are smaller by a factor T_e/T_i than the ionic terms, if $z_e \lesssim 1$, so that

$$1 + \sum_{i = 1,2} \frac{\omega_{pi}^2}{k^2 v_i^2}[1 + i\sqrt{\pi} z_i w(z_i)] = 0. \tag{2}$$

When $u < v_{1,2}$, Eq. (2) has only strongly damped solutions ($\gamma_k < 0$, $|\gamma_k| \sim \omega_k$). Instability sets in if

$$u = |\vec{u}_1 - \vec{u}_2| \gtrsim v_i,$$

and the oscillations building up most rapidly are the short-wave ones with $k \sim \omega_{pi}/v_i$ and $\gamma_k \sim \omega_k \sim \omega_{pi}$. Cerenkov interaction of both ions and electrons with these oscillations leads to the ion heating.

2. For a plasma with $T_e \ll T_{1,2}$, it is possible to neglect the electronic term in the sum over α in (1) if $\cos^2\nu \ll m_e/m_i$ and $|z_e| \gg 1$. In this case (1) coincides with (2), provided we add to (2) the term $\omega_{pe}^2/\omega_{He}^2$. When $T_1 \sim T_2$ and $u < v_i$ this equation has only strongly damped solutions, but if $u \gtrsim v_i$ the oscillations are unstable, with $\gamma_k \sim \omega_k \sim \omega_{pi}$ if $k \sim \omega_{pi}/v_i$ and $\omega_{pe} \lesssim \omega_{He}$, and with $\gamma_k \sim \omega_k \sim \sqrt{\omega_{He}\omega_{Hi}}$ if $k \sim \sqrt{\omega_{He}\omega_{Hi}}/v_i$ and $\omega_{pe} \gtrsim \omega_{He}$.

In the case under consideration, the electrons take a much smaller part in the oscillations, and the influence of the nonlinearity in the equations of motion of the electrons turns out to be weaker than in the case of electron-sound oscillations. The nonlinear wave interaction limits W to values on the order of $W \sim \sqrt{m_e/m_i}\, n_e T_i$, and the heating time turns out to be very short, $\tau \sim \sqrt{m_e/m_i}/\omega_{Hi}$.

3. If one species of ions is hot and the other species of ions and the electrons are cold ($T_2 \gg T_1$, T_e), then oscillations (which can be called ion-ion sound) exist in such a plasma when $\cos^2\nu < m_e/m_i$, and for these oscillations $|z_2| \ll 1$, $|z_1| \gg 1$, and $|z_e| \gg 1$. The frequency and the damping decrement of these oscillations are

$$\omega_k = ku_1 \pm \left(1 + \frac{\omega_{pe}^2}{\omega_{He}^2} + \frac{\omega_{p2}^2}{k^2 v_2^2}\right)^{1/2} \omega_{p1}. \qquad (3)$$

$$\gamma_k = \sqrt{\frac{\pi}{8}}\, \frac{\omega_{p2}^2}{\omega_{p1}^2}\, \frac{(\omega_k - ku_1)^3}{k^3 v_2^3}\, (ku_2 - \omega_k). \qquad (4)$$

It follows from (4) that ion-ion sound is unstable also when $u \ll v_2$. The nonlinear theory of this instability and of the heating of the hot ion component is analogous to the nonlinear theory of electron-sound instability [1-3].

Since only ions take part in the oscillations in question, the growth of W is limited whenever the nonlinear term $(\vec{v}\nabla)\vec{v}$ in the equations of motion of the ions is of the order of $\partial\vec{v}/\partial t$, i.e., when $v \sim \omega/k \sim u$. We then obtain for the oscillation energy $W \sim n_2 m_i v^2 \sim n_2 m_i u^2$.

Let us consider the heating of the ions under the influence of ion-ion sound pulsations excited by the ion current in a plasma situated in the field of a low-frequency wave with frequency $\sim\omega_{Hi}$. The diffusion of the hot ions in velocity space is determined by the quasi-linear equation for the background distribution function [1-3]

$$\partial f/\partial t = (1/v_\perp)(\partial/\partial v_\perp)(D/v_\perp)(\partial f/\partial v_\perp), \qquad (5)$$

where $v_\perp$ is the ion velocity component perpendicular to $\vec{H}_0$. (A derivation of (5) is given in [3].) The diffusion coefficient D is of the order of $D \sim \omega_{Hi} v_2^5 W/n_2 T_2$. Using the foregoing estimate for W, we obtain from (5) an estimate $\tau \sim 1/\omega_{Hi}$ for the time necessary to heat to a final temperature $T_2 \sim m_i u^2$.

[1] V. L. Sizonenko, K. N. Stepanov, and J. Teichmann, Compte Rendu du Colloque International sur l'Interaction des Champs H.F. Associes a un Champ Magnetique Statique avec une Plasma, Saclay, 15-18 January 1968.

[2] V. I. Aref'ev, I. A. Kovan, and L. I. Rudakov, ZhETF Pis. Red. 7, 286 (1968) [JETP

Lett. 7, 223 (1968)].
[3] V. L. Sizonenko and K. N. Stepanov, Physicotech. Inst. Ukr. Acad. Sci., Preprint 218, 1968.
[4] V. L. Sizonenko and K. N. Stepanov, Nuclear Fusion 7, 131 (1967).
[5] R. Z. Sagdeev, in: Voprosy teorii plazmy (Problems of Plasma Theory), No. 4, Atomizdat, 1964, p. 20.
[6] B. B. Kadomtsev, in: Fizika plazmy i problema upravlyaemykh termoyadernykh reaktsii (Plasma Physics and the Problem of Controlled Thermonuclear Reactions), v. 4, AN SSSR, 1958, p. 364.
[7] K. N. Stepanov, Zh. Tekh. Fiz. 34, 2146 (1964) [Sov. Phys.-Tech. Phys. 9, 1653 (1965)].

LOCALIZED PLASMONS AND PLASMA-EXCITON RESONANCE

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As is well known, interband electron transitions greatly influence plasma oscillations [1]. In light elements it is possible to disregard transitions of electrons from the internal shells of the atoms, since their frequencies are high compared with the plasma frequencies. However, for elements with large atomic numbers these frequencies, generally speaking, are comparable. Usually an electron excited from an internal shell goes over into a current state, i.e., the excitation spectrum of the ionic core is continuous. In some cases, however, one can expect this spectrum to begin with discrete levels of relatively small width. This pertains, for example, to dilute solutions of transition elements, when the electrons from the internal shells of the magnetic ions can be excited at free d-levels of the same ion. Owing to the possible resonance between the frequencies of such transitions and the frequencies of the plasma oscillations of the solvent, there can occur in the system localized plasma oscillations.

A similar situation can arise in strongly doped semiconductors if impurities with deep electron levels are introduced in them besides the impurities with shallow levels. At temperatures such that the degree of ionization of the deep levels is low, transitions of electrons between the levels of the discrete spectrum of the impurities are possible. By suitable choice of the carrier density it is possible to satisfy the optimal conditions for the existence of local plasma levels. The occurrence of localized plasmons, as well as the occurrence of localized optical phonons considered in [2], can be attributed to a local spike of the crystal polarizability near the impurity atoms.

If we regard the excited state of the impurity atom as a localized exciton, we can write the Hamiltonian of the system in the form

$$H = \sum_{p\sigma} E_p a^*_{p\sigma} a_{p\sigma} + 2\pi V \sum_{k\neq 0} \frac{\rho^*_k \rho_k}{k^2} + \omega_0 b^* b - 4\pi e \sum_{k\neq 0} \frac{g_k \rho_k}{k^2}(b^* + b) \tag{1}$$

$$\rho_q = e/V \sum_{k\sigma} a^*_{k\sigma} a_{k-q\sigma}.$$

Here $a^*_{p\sigma}$ and $a_{p\sigma}$ - operators of creation and annihilation of an electron with momentum p and spin σ and b^* and b - exciton operators. The charge e, generally speaking, is renor-