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LOCALIZED PLASMONS AND PLASMA-EXCITON RESONANCE

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As is well known, interband electron transitions greatly influence plasma oscillations [1]. In light elements it is possible to disregard transitions of electrons from the internal shells of the atoms, since their frequencies are high compared with the plasma frequencies. However, for elements with large atomic numbers these frequencies, generally speaking, are comparable. Usually an electron excited from an internal shell goes over into a current state, i.e., the excitation spectrum of the ionic core is continuous. In some cases, however, one can expect this spectrum to begin with discrete levels of relatively small width. This pertains, for example, to dilute solutions of transition elements, when the electrons from the internal shells of the magnetic ions can be excited at free d-levels of the same ion. Owing to the possible resonance between the frequencies of such transitions and the frequencies of the plasma oscillations of the solvent, there can occur in the system localized plasma oscillations.

A similar situation can arise in strongly doped semiconductors if impurities with deep electron levels are introduced in them besides the impurities with shallow levels. At temperatures such that the degree of ionization of the deep levels is low, transitions of electrons between the levels of the discrete spectrum of the impurities are possible. By suitable choice of the carrier density it is possible to satisfy the optimal conditions for the existence of local plasma levels. The occurrence of localized plasmons, as well as the occurrence of localized optical phonons considered in [2], can be attributed to a local spike of the crystal polarizability near the impurity atoms.

If we regard the excited state of the impurity atom as a localized exciton, we can write the Hamiltonian of the system in the form

$$H = \sum_{p\sigma} E_p a^*_{p\sigma} a_{p\sigma} + 2\pi V \sum_{k\neq 0} \frac{\rho^*_k \rho_k}{k^2} + \omega_0 b^* b - 4\pi e \sum_{k\neq 0} \frac{g_k \rho_k}{k^2}(b^* + b) \qquad (1)$$

$$\rho_q = e/V \sum_{k\sigma} a^*_{k\sigma} a_{k-q\sigma}.$$

Here $a^*_{p\sigma}$ and $a_{p\sigma}$ - operators of creation and annihilation of an electron with momentum p and spin σ and b^* and b - exciton operators. The charge e, generally speaking, is renormalized to take into account the bare dielectric constant of the crystal. g_k is the matrix element of the operator exp(ikr) between the excited and ground states of the impurity atom.

The spectrum of the plasma oscillations is determined from the poles of the retarded Green's function

$$\langle\langle \rho_q | L \rangle\rangle = -i\,\theta(t)\langle \rho_q(t) L - L \rho_q(t) \rangle, \tag{2}$$

where

$$L = \sum_{pp'\sigma\sigma'} a^*_{p\sigma} a_{p'\sigma'},$$

$\theta(t)$ is the step function. To find $\langle\langle p_q | L \rangle\rangle$, an equation of motion is set up for the Green's function $\langle\langle a^*_{p\sigma} a_{p-q\sigma} | L \rangle\rangle$. The higher Green's functions contained in this equation, made up only of electron operators, separate and only the terms that diverge as $q \to 0$ remain (this approximation is equivalent to a summation of ladder diagrams):

$$\sum_r \frac{1}{r^2} \langle\langle \rho_r a^*_{p-r\sigma} a_{p-q\sigma} | L \rangle\rangle = 1/q^2\, n_{p-q} \langle\langle \rho_q | L \rangle\rangle .$$

Equations of motion are also set up for the Green's functions that include the exciton operators, and the Green's functions of higher order with respect to them are separated by pairing the Fermi operators. Here again, only the terms with singularities at small momentum transfers are retained. By summing the Green's functions

$$\langle\langle a^*_{p\sigma} a_{p-q\sigma} | L \rangle\rangle$$

over the indices p and σ we obtain the following integral equation for the determination of the sought Green's function

$$\langle\langle \rho_q | L \rangle\rangle = \left[\frac{1}{\epsilon_0(q\omega)} - 1\right]\left[\frac{q^2}{4\pi e} + \frac{8\pi g^*_q \omega_0}{(\omega^2-\omega_0^2)V} \sum_k \frac{g_k \langle\langle \rho_k | L \rangle\rangle}{k^2}\right], \tag{3}$$

where $\epsilon_0(k\omega)$ is the dielectric constant of the unperturbed electron gas in the random-phase approximation:

$$\epsilon_0(k\omega) = 1 + \frac{4\pi e^2}{k^2 V} \sum_{p\sigma} \frac{n_{p-k} - n_p}{\omega - E_{p-k} + E_p} . \tag{4}$$

The solution of (3) shows that, besides the volume plasmons, whose frequencies are determined from the vanishing of $\epsilon_0(k\omega)$, there can exist local or quasilocal plasmons. Their frequency is determined from the equation

$$\frac{8\pi \omega_0 e^2}{(\omega^2-\omega_0^2)V} \sum_k \frac{|g_k|^2}{k^2}\left[\frac{1}{\epsilon_0(k\omega)} - 1\right] = 1. \tag{5}$$

By quasilocal levels, as usual, we mean those solutions of (5) which fall in the continuous spectrum and therefore have a finite width. It should be noted, however, that owing to the complex character of $\epsilon_0(k\omega)$, solutions lying outside the continuous spectrum, i.e., true local levels, also have a finite width.

It is easy to formulate the conditions for the existence of the local plasmon level. For example, for the dipole transition $eq_k = ikd$, where d is the dipole moment of the impurity atom. The limiting value of d, at which the level occurs, is determined from (5), in which we set ω equal to the Langmuir frequency ω_p. The integral in the right side of (5) converges when $\omega = \omega_p$, and therefore at a specified difference $\omega_0 - \omega_p$ the local level appears only starting with a certain critical value $d = d_c$. This is a manifestation of the general regularity observed by I. M. Lifshitz [3] in the spectra of three-dimensional systems. The quantity d_c tends to zero as ω_0 approaches ω_p. However, in practice the conditions for the occurrence of local plasmons turn out to be less favorable, owing to the finite width of the electronic levels of the impurity atoms, which was not taken into account here. The widths of the d-levels of the atoms of transition elements dissolved in other metals are determined, in accordance with [4], by their mixing with the levels of the conduction electrons of the metal, and can be quite appreciable if the d-level is close to the Fermi surface.

The effect considered above is a resonance of plasma oscillations with the localized exciton. There exist, however, materials in which plasmon resonance with volume excitons is possible; these are compounds of transition elements, in which the Frenkel exciton is realized as an excited state of the electrons of the unfilled d-shells. Such excitons have been recently investigated actively in insulating compounds of transition elements (see, e.g., [5]). Yet there is a large group of semimetallic compounds of transition elements with "innate" carrier densities $\sim 10^{18} - 10^{20}$ cm^{-3}, wherein the number of carriers can be varied in a sufficiently wide range by doping. The plasma frequency amounts in this case to several tenths of an eV, and therefore the plasmons are clearly defined elementary excitations. Since the exciton band is very narrow, the conditions for the realization of the plasma-exciton resonance are quite favorable here. By a method analogous to that used earlier, the following dispersion relations are obtained for the determination of the frequencies of the plasmons and of the excitons, respectively:

$$1 = \frac{8\pi e^2 |g_q|^2 \omega_q}{q^2(\omega^2 - \omega_q^2) v_0} \left[\frac{1}{\epsilon_0(q\omega)} - 1 \right], \tag{6}$$

$$\omega = \omega_q + \frac{4\pi e^2 |g_q|^2}{q^2 v_0} \left[\frac{1}{\epsilon_0(q\omega)} - 1 \right], \tag{7}$$

where ω_k is the frequency of the exciton with momentum k, and v_0 is the volume of the unit cell. At exact resonance between the bare plasma and exciton frequencies, the line shift is of the order of g_{qi} and not $|g_q|^2$.

In conclusion, it should be indicated that the resonances discussed above can be realized also with participation of surface excitons and plasmons.

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POSSIBLE SEARCH FOR $K^+ \to \pi^+ + \nu + \nu$ DECAYS

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In the customary theory [1] it is assumed that the Lagrangian of the weak interaction contains the product of only charge currents. This limitation is connected with the fact that experiments have not revealed decays of strange particles of the type

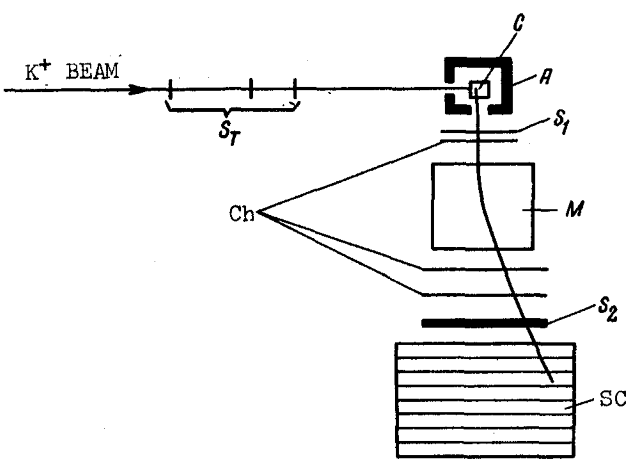
$$K^0 \to \mu^+ + \mu^-, \tag{1}$$

$$K^+ \to \pi^+ + e^+ + e^-, \text{ etc.} \tag{1b}$$

The decays (1), however, are possible in second order weak-interaction perturbation theory. The application of current-algebra methods to them has led to an unexpected conclusion (see [2]) that the weak interactions conserve the current-current form only up to relatively small momentum values.

This result may signify that the weak interaction has a much more complicated structure and is represented in the form of the product of currents only in the low-energy limit. Examples of such more complicated schemes of weak interaction are two models of the renormalizable interaction, proposed respectively by Tanikawa-Wanatabe [3] and Kummer-Segre [4]. Investigations have shown (see [5,6]) that these models agree with modern data, and whereas in the former model (T-W) the agreement between the vector constants of the neutron and muon decay is accidental, in the second model the equality of these constants is predicted by the theory [6].

An interesting feature of the renormalizable models is that transitions with change of strangeness and with emission of two leptons, i.e., of type (1), are forbidden in them (in the lowest order of perturbation theory), but transitions with emission of a neutron pair, i.e., of the type



$$K^+ \to \pi^+ + \nu + \bar{\nu} \tag{2}$$

are allowed, although they may be somewhat suppressed. We note that in the case of the second model, according to [6], the possibility of an appreciable suppression of emission of $\nu\bar{\nu}$ compared with emission of the $\ell^{\pm}\nu$ pair is not excluded.