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ANGULAR ANISOTROPY OF THE FISSION OF Pb^{204} AND Pb^{208} BY ALPHA PARTICLES NEAR THE THRESHOLD

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Recently published results of the measurement of the angular anisotropy of the fission of Pb^{206} by alpha particles [1] have established that the energy gap in the spectrum of the internal excitations of the nucleus Po^{210} in the transition state is anomalously large, $2\Delta_f = 4$ MeV. This is more than double the value $2\Delta_g = 1.5$ for the equilibrium ground state, in which the Po^{210} nucleus is close to spherical. The contradictory character of the information [1-4] on the dependence of the parameter Δ , which determines the role of the effect of paired nucleon correlation, on the nuclear deformation has stimulated us to perform similar measurements for the other even-even isotopes of lead, Pb^{204} and Pb^{208} .

The experiments were performed with the cyclotron of our institute, using a track procedure for registering the fission fragments. The experiments were so organized that simultaneous measurements could be made of the fission cross section σ_f and the angular distribution of the fragments $W(\theta)$. The alpha-particle energies were decreased to 29 - 38 MeV with the aid of aluminum foils. The accuracy with which the alpha-particle energy was measured was not worse than ± 0.2 MeV. The targets were prepared by an electrolytic method from separated Pb^{204} and Pb^{208} . The upper limit of the extraneous strongly-fissioning impurities was less than 10^{-6} %, as estimated in reactor experiments, and guaranteed background-free measurements in the entire investigated energy range.

The results of the measurements of the angular anisotropy $W(0^\circ)/W(90^\circ)$ of the fission of Po^{208} , Po^{212} , and Po^{210} are shown in the figure as functions of the excitation energy in the transition state $E^* = E_x - E_f$, where E_x is the initial excitation energy of the compound nuclei and E_f is the height of the fission barrier. The values of E_f for Po^{208} , Po^{210} , and Po^{212} were assumed equal to 19.8, 20.5, and 18.8 MeV, respectively (the last two according to [5], and for Po^{208} from the measured course of $\sigma_f(E_x)$).

A comparison of the data shown in the figure reveals a striking difference in the energy dependence of $W(0^\circ)/W(90^\circ)$ near threshold namely a sharp growth in the case of Po^{210} , and constancy in the case of Po^{208} and Po^{212} , similar to that observed for Tl^{201} and At^{213} in the (α, f) reaction on gold in bismuth [6]. This discrepancy between the results of [1] and the aggregate of the experimental data for other nuclei may possibly be the consequence of an individual peculiarity of the Po^{210} nucleus and deserves a special study. All that matters to us here is that these difference in the course of the angular anisotropy should influence our ideas concerning the value of Δ_f .

An estimate of the energy gap $2\Delta_f$ in [1] was based on the following considerations. In

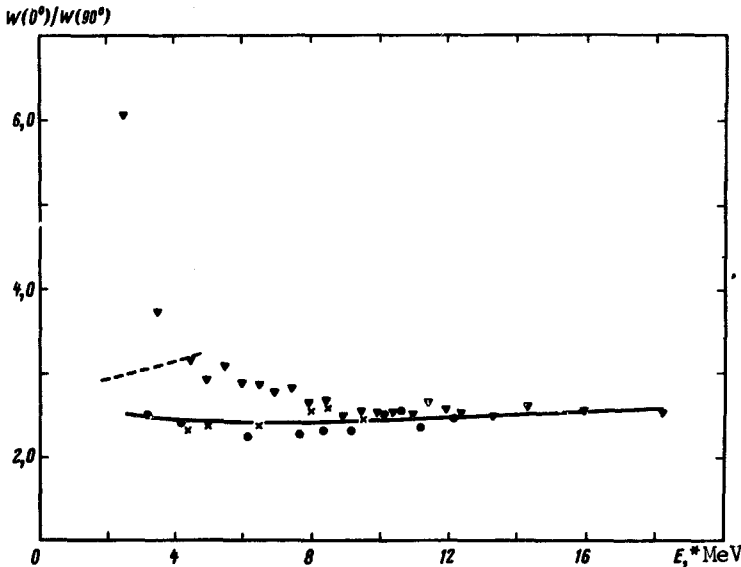
the vicinity of $E^* = 2\Delta_f$, the mean-squared projection of the angular momentum l on the symmetry axis of the nucleus, $K_0^2 = \overline{K^2}$, experiences a jumplike increase in the two-quasiparticle value $K_0^2 = 2\overline{K^2} \approx 10 - 14$ in accordance with [7],

$$W(0^\circ)/W(90^\circ) = \frac{2}{3} p^{3/2} / \int_0^p x^{1/2} e^{-x} I_0(x) dx = 1 + \frac{3}{5} p + \dots ;$$

$$p = \frac{\overline{l^2}}{2K_0^2} .$$

This leads to an appreciable increase of the angular anisotropy inside the energy gap. The value of $W(0^\circ)/W(90^\circ)$ corresponding to $K_0^2 = 2\overline{K^2}_{cr} = 12$ in the reaction $Pb^{206}(\alpha, f)$ is shown dashed in the figure. The point of intersection of the dashed curve with the experimental one at $E_{cr} \approx 4$ MeV was indeed used in [1] for the estimate of $2\Delta_f$. All the values of $W(0)/W(90^\circ)$ for the reaction $Pb^{204}(\alpha, f)$, investigated in practically the same range of E^* , lie below the dashed curve.

This discrepancy stimulates a search for other ways of estimating Δ_f . A method with greater freedom from uncertainties and individual peculiarities of the variation of K_0^2 in regions of $E_x = E_f$ that are narrow and difficult to investigate was realized in [3, 4, 8]. It consists of estimating the energy of the phase transition from the superfluid state into the Fermi-gas state, $E_{cr} = 0.48 a_f \Delta_f^2$, which serves as a measure of the deviations of $K_0^2 = J_{eff}/\hbar^2$ from the Fermi-gas model. When $E^* < E_{cr}$ we have $K_0^2 = C\sqrt{E^*}$. The solid plot of $W(0^\circ)/W(90^\circ)$ in the figure corresponds to this dependence of K_0^2 on E^* with a coefficient $C = J_{eff}/\hbar^2 a_f^{1/2} = 10.5 \text{ MeV}^{-1/2}$, calculated under the assumption that the level-density parameter of the transition states is $a_f = A/8$, and the effective moment of inertia of the nucleus for J_{eff} is equal to its rigid-body value in the liquid-drop model with $(Z^2/A)_{crit} = 46$ [9].



Fission angular anisotropy vs. excitation energy in the transition state of $Po^{208}(\bullet)$, $Po^{210}(\blacktriangledown)$, and $Po^{212}(\times)$. The curves are explained in the text.

The entire set of values of $W(0^\circ)/W(90^\circ)$, including those for Po^{210} [1], agree well, at least up to 10 MeV, with the Fermi-gas relation, and by the same token with the published [4, 8, 10] estimates $E_{cr} = 10 - 16$ MeV. When $a_f = A/8$, these values of E_{cr} correspond to $\Delta_f = 0.8$ to 1.1 MeV, which are close to Δ_g for the ground state.

If we substitute $\Delta_f = 2$ MeV in the expression for E_{cr} and make the same choice of a_f , which is confirmed by many data, then we obtain the unreasonably high value $E_{cr} = 50$ MeV.

Thus, the results of our experiment and of our analysis confirm

the opinion that Δ_f has a weak dependence on the deformation of the nucleus in the fission process [2, 4].

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CAUSALITY AND SCATTERING OF WAVE PACKETS

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1. An attractive possibility of solving the fundamental difficulties with the divergences within the framework of the theory with pseudo-Hermitian Hamiltonian was proposed in recent papers [1,2]. If this pseudo-Hermitian operator satisfies certain additional conditions, then the S matrix for physical particles (states with positive metric) turns out to be unitary [1]. The question of causality within the framework of this theory, however, remains moot.

2. Let us consider the space-time picture of the scattering of wave packets [3]. For simplicity we confine ourselves to elastic scattering in the s-state in the relativistic region, so that $E = k$.¹⁾ At sufficiently large $r > R$, we represent the vector of the wave-packet scattering in the form

$$\psi(r, t) = r^{-1} \{ \phi_{in}(r, t) + \phi_{out}(r, t) \}, \quad (1)$$

$$\phi_{in}(r, t) = \int_0^{\infty} C(k) \exp[-ik(r+t)] dk, \quad (1a)$$

$$\phi_{out}(r, t) = \int_0^{\infty} C(k) \exp[2i\delta(k) + ik(r-t)] dk, \quad (1b)$$

where $C(k)$ is the distribution of the wave packets with respect to the energy (momentum), and $\delta(k)$ is the scattering phase shift. The classical concept of stability stipulates that when

¹⁾The notation is that of [1].