the opinion that Δ_f has a weak dependence on the deformation of the nucleus in the fission process [2, 4].

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[1] L. G. Moretto, R. S. Gatti, S. G. Thompson, J. R. Huizenga, and J. O. Rasmussen, Phys. Rev. 178, 1845 (1969).
[2] V. M. Strutinski and V. A. Pavlinchuk, Physics and Chem. of Fission (Proc. Symp. Salzburg 1965), Vienna, 1, 127 (1965); Yu. T. Grin' and V. M. Strutinskii, Yad. Fiz. 1, 420 (1965) [Sov. J. Nuc. Phys. 1, 299 (1965)].
[3] J. J. Griffin, Phys. Rev. 132, 2204 (1963).
[4] G. N. Smirenkin, V. G. Nesterov, and A. S. Grishin, Yad. Fiz. 6, 921 (1967) [Sov. J. Nuc. Phys. 6, 671 (1968)].
[5] S. G. Thompson, Arkiv fys. 36, 267 (1967).
[6] R. Chaudhry, R. Vandenbosch, and J. R. Huizenga, Phys. Rev. 126, 220 (1962).
[7] I. Halpern, V. M. Strutinski, Proc. Second UN Internat. Conf., Geneva, 15, 408 (1958).
[8] J. E. Gindler, G. L. Bate, and J. R. Huizenga, Phys. Rev. 136, 1333 (1964).
[9] V. M. Strutinskii, Yad. Fiz. 1, 821 (1965) [Sov. J. Nuc. Phys. 1, 588 (1965)]; K. G. Kuvatov, V. N. Okolovich, and G. N. Smirenkin, ZhETF Pis. Red. 8, 277 (1968) [JETP Lett. 8, 171 (1968)].
[10] A. V. Ignatyuk and Yu. N. Shubin, Yad. Fiz. 8, 1135 (1968) [Sov. J. Nuc. Phys. 8, 660 (1969)].

CAUSALITY AND SCATTERING OF WAVE PACKETS

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1. An attractive possibility of solving the fundamental difficulties with the divergences within the framework of the theory with pseudo-Hermitian Hamiltonian was proposed in recent papers [1,2]. If this pseudo-Hermitian operator satisfies certain additional conditions, then the S matrix for physical particles (states with positive metric) turns out to be unitary [1]. The question of causality within the framework of this theory, however, remains moot.

2. Let us consider the space-time picture of the scattering of wave packets [3]. For simplicity we confine ourselves to elastic scattering in the s-state in the relativistic region, so that E = k.[1] At sufficiently large r > R, we represent the vector of the wave-packet scattering in the form

$$\psi(r,t) = r^{-1}\{ \phi_{in}(r,t) + \phi_{out}(r,t) \}, \tag{1}$$

$$\phi_{in}(r,t) = \int_0^\infty C(k)\exp[-ik(r+t)]\,dk, \tag{1a}$$

$$\phi_{out}(r,t) = \int_0^\infty C(k)\exp[2i\delta(k) + ik(r-t)]\,dk, \tag{1b}$$

where C(k) is the distribution of the wave packets with respect to the energy (momentum), and δ(k) is the scattering phase shift. The classical concept of stability stipulates that when

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[1] The notation is that of [1].

$r > t$ the value of $\phi_{out}(r, t)$ for localized packets should be negligibly (exponentially) small (there should be no anticipation effect). It is shown in [3] that in the theory with a Hermitian Hamiltonian, in accord with the principle, for scattering in the resonance region,

$$\{\exp[2i\delta(k)] = (k - m + \frac{1}{2} i\gamma)/(k - m - \frac{1}{2} i\gamma)\}\ \phi_{out}(r, t)$$

is exponentially small when $r > t$, and $\phi_{out}(r, t)$ gives a delay with the lifetime of the resonance when $r < t$. On the other hand, the theory with a pseudo-Hermitian Hamiltonian, for scattering in the region of the unstable "ghost resonance

$$\{\exp[2i\delta(k)] = (k - m - \frac{1}{2} i\gamma) / (k - m + \frac{1}{2} i\gamma)\}$$

with the special choice

$$C(k) = \frac{1}{2}(\frac{\Delta}{\pi})^{3/2}[(k - m)^2 + \Delta^2]^{-1}$$

they obtained in [1] for $r > R$

$$\begin{cases} \phi_{in}(r, t) = \frac{1}{2}(\frac{\Delta}{\pi})^{1/2} \exp[-\Delta\,|r + t|\,] \equiv \phi^L_{in} & \text{(2a)} \\ \phi_{out}(r, t) = \frac{1}{2}(\frac{\Delta}{\pi})^{1/2}\{\frac{2\Delta + \gamma}{2\Delta - \gamma}\exp[-\Delta(r - t)] - \end{cases}$$

$$- \frac{8\gamma\Delta}{4\Delta^2 - \gamma^2}\exp[-\frac{1}{2}(r - t)]\} \equiv \phi^L_{out}\ . \qquad r > t \qquad \text{(2b)}$$

The presence in (2b) of a second term with $\exp[-\gamma(r - t)/2]$ was interpreted as violation of causality (macrocausality in the case $r \gg t$). It was stated in [1] on the basis of (2) that the "anticipation" of the center of the packet

$$< r >_{ont} = 4\pi \int_0^\infty r^2 |\phi_{out}(r, t)|^2 dr$$

is smaller than the packet dimensions $\sim\Delta^{-1}$, so that the causality violation is not noticeable for the centers of the packet. For the choice of $C(k)$ used in [1], this statement is valid, since a more exact calculation yields ($r > R$, $r > t$, $r - t > m^{-1}$):

$$\phi_{out}(r, t) = \phi^L_{out} \exp[im(r - t)] - \frac{1}{2i}(\frac{\Delta}{\pi})^{3/2} \frac{m - \frac{1}{2} i\gamma}{m + \frac{1}{2} i\gamma} \frac{1}{m^2 + \Delta^2} \frac{1}{r - t} \qquad (3)$$

and consequently $<r>_{out}$ does not exist, and an essentially different choice of $C(k)$ is necessary in order for a result close to [1] to be valid.

3. We shall show, however, that a similar and even more apparent "violation" of causality may occur in the usual theory with a Hermitian Hamiltonian, provided account is taken of the existence of inelastic processes. Let us assume first that there are no inelastic processes in principle, and then $f(k)$, the scattering amplitude (or its derivatives) becomes discontinuous only at $k = 0$. Then, taking into account the non-exponential terms [4], when

r >> t:

$$\phi_{in}(r,t) \sim a_n|r+t|^{-n}, \quad \phi_{out}(r,t) \sim b_n(r-t)^{-n-3/2} \quad r>>t, \tag{4}$$

where n is the order of the derivative $d^nC(k)/dk^n$ differing from zero when k = 0, and a_n and b_n are determined by the behavior of C(k) and δ(k) when k = 0. It is seen from (4) that the presence of a nonzero term in $\phi_{out}(r, t)$ when r > t does not mean violation of causality, since $\phi_{in}(r, t)$ has a still larger term. On the other hand, if there exists an inelastic process with a threshold $k = k_{thr} > 0$, then f(k) (on the basis of the optical theorem) has a discontinuity (or discontinuous derivatives) at $k = k_{thr} > 0$, where C(k) is continuous. Then, taking the non-exponential terms into account,

$$\phi_{in}(r,t) \sim a_n|r+t|^{-n}, \quad \phi_{out}(r,t) \sim d_n(r-t)^{-3/2} \quad r>>t, \tag{5}$$

where a_n is determined by C(k) with k = 0, and d_n is determined by the behavior of C(k) and f(k) (δ(k)) when $k = k_{thr}$. It follows from (5) that causality is "violated" within the framework of the usual theory with a Hermitian Hamiltonian much more at macroscopic scales r >> t than in the theory with a non-Hermitian Hamiltonian. It is important to emphasize that this effect of causality"violation" takes place only when an inelastic process unique to quantum theory is taken into account, namely the transformation of certain particles into others.

Detailed proofs, examination of inelastic processes, and estimates of the possibility of observing (enhancing) the "violation" of causality will be published separately.

After completing this paper, I received a preprint by T. D. Lee [5], in which a power-law "violation" of causality is considered within the framework of a model with a pseudo-Hermitian Hamiltonian. I am grateful to Professor Lee for the opportunity of becoming acquainted with his work prior to publication.

[1] T. D. Lee and G. C. Wick, Nuclear Physics B9, 209 (1969)
[2] T. D. Lee, Proc. 1969 Topical Conf. on Weak Interactions (CERN).
[3] E. P. Wigner, Phys. Rev. 28, 145 (1955); M. Goldberger and K. Watson, Collision Theory, Wiley, 1964.
[4] L. A. Khalfin, Dokl. Akad. Nauk SSSR 115, 277 (1957) [Sov. Phys.-Dokl. 2, 340 (1958)]; Zh. Eksp. Teor. Fiz. 33, 1371 (1958) [Sov. Phys.-JETP 6, 1053 (1959)]; Quantum Theory of the Decay of Physical Systems, Candidate's Dissertation, FIAN, 1960.
[5] T. D. Lee, A Relativistic Complex Pole Model with Indefinite Metric, Preprint, 1969.

INSTABILITY OF A WEAKLY-INHOMOGENEOUS PLASMA WITH TWO SPECIES OF IONS IN THE ABSENCE OF A MAGNETIC FIELD

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1. B. B. Kadomtsev and A. V. Nedospasov [1] have shown that the positive column of a gas discharge is unstable in external parallel electric and magnetic fields. We shall show in this paper that in a plasma containing two species of ions instability is possible also in the absence of a magnetic field, if the electron mobility depends on the coordinates. This can be caused by the coordinate dependence of the effective electron temperature T_-. This excites quasineutral density and field oscillations. In the approximation where the electron