r >> t:

$$\phi_{in}(r,t) \sim a_n|r + t|^{-n}, \quad \phi_{out}(r,t) \sim b_n(r-t)^{-n-3/2}\ r >> t, \tag{4}$$

where n is the order of the derivative $d^nC(k)/dk^n$ differing from zero when k = 0, and $a_n$ and $b_n$ are determined by the behavior of C(k) and δ(k) when k = 0. It is seen from (4) that the presence of a nonzero term in $\phi_{out}(r, t)$ when r > t does not mean violation of causality, since $\phi_{in}(r, t)$ has a still larger term. On the other hand, if there exists an inelastic process with a threshold $k = k_{thr} > 0$, then f(k) (on the basis of the optical theorem) has a discontinuity (or discontinuous derivatives) at $k = k_{thr} > 0$, where C(k) is continuous. Then, taking the non-exponential terms into account,

$$\phi_{in}(r,t) \sim a_n|r+t|^{-n}, \quad \phi_{out}(r,t) \sim d_n(r-t)^{-3/2}\ r >> t, \tag{5}$$

where $a_n$ is determined by C(k) with k = 0, and $d_n$ is determined by the behavior of C(k) and f(k) (δ(k)) when $k = k_{thr}$. It follows from (5) that causality is "violated" within the framework of the usual theory with a Hermitian Hamiltonian much more at macroscopic scales r >> t than in the theory with a non-Hermitian Hamiltonian. It is important to emphasize that this effect of causality"violation" takes place only when an inelastic process unique to quantum theory is taken into account, namely the transformation of certain particles into others.

Detailed proofs, examination of inelastic processes, and estimates of the possibility of observing (enhancing) the "violation" of causality will be published separately.

After completing this paper, I received a preprint by T. D. Lee [5], in which a power-law "violation" of causality is considered within the framework of a model with a pseudo-Hermitian Hamiltonian. I am grateful to Professor Lee for the opportunity of becoming acquainted with his work prior to publication.

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## INSTABILITY OF A WEAKLY-INHOMOGENEOUS PLASMA WITH TWO SPECIES OF IONS IN THE ABSENCE OF A MAGNETIC FIELD

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1. B. B. Kadomtsev and A. V. Nedospasov [1] have shown that the positive column of a gas discharge is unstable in external parallel electric and magnetic fields. We shall show in this paper that in a plasma containing two species of ions instability is possible also in the absence of a magnetic field, if the electron mobility depends on the coordinates. This can be caused by the coordinate dependence of the effective electron temperature $T_-$. This excites quasineutral density and field oscillations. In the approximation where the electron

mobility gradient is small, the oscillation frequency is

$$\omega = (\mathbf{k}\,\mathbf{E}_o)\,\frac{\mu_-\mu_1 n_2 + \mu_-\mu_2 n_1 + \mu_1\mu_2 n_-}{\mu_- n_- + \mu_1 n_1 + \mu_2 n_2} \tag{1}$$

($\vec{k}$ is the wave vector, $\vec{E}_0$ the external field, $\mu_{-,1,2}$ and $n_{-,1,2}$ the mean mobilities and densities of the electrons and the ions). The general form of the instability condition is quite complicated, but simplifies when $\nabla\mu_-/\mu_- = L_H^{-1} << k$ and $n_2 << n_1$, taking the form

$$A\,\frac{eE_o}{Tk}\,\frac{1}{kL_H}\,\frac{n_2}{n_-}\left|\frac{\mu_2 - \mu_-}{\mu_2 + 2\mu_-}\right| > 1 \tag{2}$$

where A is a number smaller than unity and T is the ion temperature.

Then

$$\omega \approx (\mathbf{k}\,\mathbf{E}_o)\,\mu_2. \tag{1'}$$

2. The system of equations describing the problem is

$$\frac{\partial}{\partial t} n_- + \operatorname{div}\{-D_-\nabla n_- - \mu_- E n_-\} = (W_1 + W_2)\,n_-\,,$$

$$\frac{\partial}{\partial t} n_{1,2} + \operatorname{div}\{-D_{1,2}\nabla n_{1,2} + \mu_{1,2} E n_{1,2}\} = W_{1,2} n_-\,, \tag{3}$$

$$n_- = n_1 + n_2$$

($D_{-,1,2}$ are the diffusion coefficients of the electrons and ions, and $W_{1,2}$ are the impact-ionization coefficients). It can be shown that density inhomogeneity does not lead to instability in the absence of a magnetic field. We shall therefore consider mobility inhomogeneity in two cases.

3. If

$$\frac{\nabla\mu_{-,1,2}}{\mu_{-,1,2}} = \frac{1}{L_{H-,1,2}}$$

and $L_{H-,1,2}$ is much larger than all the system dimensions, then, linearizing Eq. (3) with respect to $n^1_{-1,2} = (n - n_0)_{-,1,2}$ and $\nabla\phi = \vec{E}_0 - \vec{E}$ ($n_0$ is the stationary density), and putting $\phi, n^1_{-1,2} \sim \exp(-i\omega t + i\int\vec{k}(\vec{r})d\vec{r})$ we get, accurate to $(kL_{H-,1,2})^{-1} << 1$

$$\omega = \mu_o(\mathbf{k}\,\mathbf{E}_o) - ik^2\,\frac{D_-D_1(n_- + n_1) + D_-D_2(n_- + n_2) + D_1D_2n_-}{D_-n_- + D_1n_1 + D_2n_2} +$$

$$+ \frac{i(\mathbf{k}\,\mathbf{E}_o)}{(\mu_1\mu_2 n_- + \mu_-\mu_1 n_2 + \mu_2\mu_- n_1)}\{n_-(\mathbf{k}\nabla\mu_-)(\mu_o - \mu_1)(\mu_o - \mu_2) + \tag{4}$$

$$+ n_1(\mathbf{k}\nabla\mu_1)(\mu_o - \mu_2)(\mu_o + \mu_-) + n_2(\mathbf{k}\nabla\mu_2)(\mu_o - \mu_1)(\mu_o + \mu_-)\},$$

where

$$\mu_o = \frac{\mu_1\mu_2 n_- + \mu_-\mu_1 n_2 + \mu_-\mu_2 n_1}{\mu_1 n_1 + \mu_2 n_2 + \mu_- n_-}.$$

In the limiting case $n_2 \ll n_1$ and $\nabla\mu_1 = \nabla\mu_2 = 0$, taking into account the fact that $\mu_- \gg \mu_{1,2}$, we obtain (1') and (2) from (4).

4. In experiments we usually deal with a gas discharge in a cylindrical tube (of radius R). In this case the solution (3) for the case when $\mu_-$ depends weakly on the radius,

$$\mu_- = \mu^o_-\left(1 - \frac{r}{L_H}\right); \quad L_H >> R$$

can be sought in the form

$$n^1_{-,1,2}(n^\vee_- \nabla\phi) \sim \Sigma C_j I_m\left(\beta_{mj}\frac{r}{R}\right)\exp(im\phi + ikz - i\omega t) \qquad (5)$$

$\beta_{mj}$ is the j-th root of the Bessel function of order m. The boundary conditions take the form $n^1_{-,1,2}(R) = 0$. It is customary [1] to confine oneself to only one term of the series (5). In the presence of a magnetic field, this suffices to produce in the imaginary part of the frequency terms that describe buildup of oscillations. It can be shown that such an approximation is insufficient without a magnetic field, for in such an approximation the imaginary part of the frequency has no terms due to the dependence of $\mu_-$ on r. It is therefore necessary to retain at least the first two terms of the series (5). Substituting (5) in (3), multiplying by $rI_m[\beta_{mj}(r/R)]dr$, and integrating from 0 to R at j = 1 and 2, we obtain a system of equations for the constants $C_j$. Equating the determinant of the system to zero, we get for the frequency an expression coinciding with (1), and for the instability condition (when $n_2 \ll n_2$) we get

$$A\frac{eER}{T}\frac{R}{L_H}\frac{n_2}{n_-}\left|\frac{\mu_2-\mu_1}{\mu_2+2\mu_1}\right| > 1 \qquad (6)$$

$$A \approx \frac{1}{20} \text{ if } m = 0, \quad A \approx \frac{1}{40} \text{ if } |m| = 1.$$

As seen from (1) or from (6), there is no instability when $n_2 \to 0$ or $\mu_2 \to \mu_1$. We note that, unlike in [1], instability is possible also when m = 0.

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## SINGULARITIES OF THE RESISTIVE STATE WITH CURRENT IN THIN SUPERCONDUCTING FILMS

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At temperatures below critical, a thin superconducting film is capable of carrying a nondissipative current, whose density is lower than a certain critical value, $j < j_c$. When $j > j_c$, the film has resistance, which corresponds in first approximation to Ohm's law for the