where

$$\mu_0 = \frac{\mu_1\mu_2 n_- + \mu_-\mu_1 n_2 + \mu_-\mu_2 n_1}{\mu_1 n_1 + \mu_2 n_2 + \mu_- n_-} .$$

In the limiting case $n_2 \ll n_1$ and $\nabla\mu_1 = \nabla\mu_2 = 0$, taking into account the fact that $\mu_- \gg \mu_{1,2}$, we obtain (1') and (2) from (4).

4. In experiments we usually deal with a gas discharge in a cylindrical tube (of radius R). In this case the solution (3) for the case when μ_- depends weakly on the radius,

$$\mu_- = \mu^\circ_-\left(1 - \frac{r}{L_H}\right); \quad L_H \gg R$$

can be sought in the form

$$n^1_{-,1,2}(n^\circ_- \nabla \phi) \sim \Sigma C_j I_m\left(\beta_{mj}\frac{r}{R}\right)\exp(im\phi + ikz - i\omega t) \tag{5}$$

β_{mj} is the j-th root of the Bessel function of order m. The boundary conditions take the form $n^1_{-,1,2}(R) = 0$. It is customary [1] to confine oneself to only one term of the series (5). In the presence of a magnetic field, this suffices to produce in the imaginary part of the frequency terms that describe buildup of oscillations. It can be shown that such an approximation is insufficient without a magnetic field, for in such an approximation the imaginary part of the frequency has no terms due to the dependence of μ_- on r. It is therefore necessary to retain at least the first two terms of the series (5). Substituting (5) in (3), multiplying by $rI_m[\beta_{mj}(r/R)]dr$, and integrating from 0 to R at j = 1 and 2, we obtain a system of equations for the constants C_j. Equating the determinant of the system to zero, we get for the frequency an expression coinciding with (1), and for the instability condition (when $n_2 \ll n_2$) we get

$$A\frac{eER}{T}\frac{R}{L_H}\frac{n_2}{n_-}\left|\frac{\mu_2 - \mu_1}{\mu_2 + 2\mu_1}\right| > 1 \tag{6}$$

$$A \approx \frac{1}{20} \text{ if } m = 0, \quad A \approx \frac{1}{40} \text{ if } |m| = 1.$$

As seen from (1) or from (6), there is no instability when $n_2 \to 0$ or $\mu_2 \to \mu_1$. We note that, unlike in [1], instability is possible also when m = 0.

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SINGULARITIES OF THE RESISTIVE STATE WITH CURRENT IN THIN SUPERCONDUCTING FILMS

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At temperatures below critical, a thin superconducting film is capable of carrying a nondissipative current, whose density is lower than a certain critical value, $j < j_c$. When $j > j_c$, the film has resistance, which corresponds in first approximation to Ohm's law for the normal metal, $j = \sigma E$. It is curious, however, that the normal state with an electric field and current ($j = \sigma E$) is stable against the occurrence of infinitesimally small nuclei of the superconducting phase, down to the very weakest fields (and currents). The point is that once a Cooper pair is produced, it is accelerated by the electric field as a unit, acquiring in the course of time an arbitrary large momentum; by virtue of the Landau mechanism, this process destroys the pairing. This circumstance can also be deduced from a linear nonstationary generalization of the Ginzburg-Landau equation for the ordering parameter (cf., e.g., [1, 2]). Thus, the "current-voltage characteristic" of a thin film has approximately the form shown in Fig. 1. On the line corresponding to Ohm's law, however, the ordering parameter exhibits fluctuations about the zero value; the smaller the field E ($T < T_C$, by assumption), the larger these fluctuations. The plot of $j(E)$ will therefore actually assume the form of the dashed curve of Fig. 1.

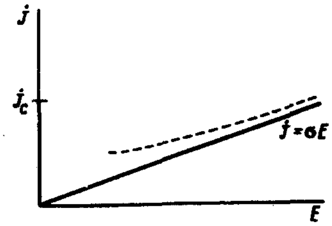

Fig. 1

The contribution of the ordering-parameter fluctuations to the conductivity in a weak field at $T > T_c$ (the so-called "paraconductivity") was first investigated by Aslamazov and Larkin [3] (see also [4]). The nonlinear dependence of the conductivity on the field (again at $T > T_c$) was investigated in [5]. Later on, Thomson [6] has shown that in ordinary alloys the results of [3] should be corrected by taking into account the so-called "anomalous terms" [1]. Deferring these complications to another paper, we wish here to demonstrate qualitatively the phenomenon represented in Fig. 1, i.e., the essentially nonlinear character of the current-voltage characteristic of a superconducting film. To this end, we confine ourselves to the model of a film with paramagnetic impurities [1] (or in a strong magnetic field), for which the role of the anomalous terms is small.

The diagram contributing the the current connected with the fluctuations is shown in Fig. 2. The solid lines correspond to the electron Green's functions, and the dashed lines correspond to averaging over the impurities, while the wavy line corresponds to the Cooper vertex part (designated $\mathcal{K}(k, \omega_n)$ in [3]). This form, unlike in [3], makes it possible to obtain directly a gauge-invariant expression for the current increment (on the thermodynamic-frequency axis; we are using only the vector-potential $\vec{A}$):

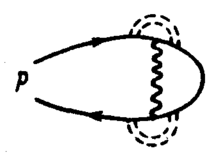

Fig. 2

$$\delta j_{\omega_0} = \frac{8\pi^2 Ne\tau}{m^2 p_0} T \sum_{\omega} \int dk \{(k - \frac{2e}{c} A)\mathcal{K}(k)\}_{\omega,\, \omega-\omega_0} \qquad (1)$$

Expression (1) should be continued analytically continued to the real axis.

By virtue of the essential nonlinearity of the problem, the function $\mathcal{K}$ includes a dependence on the electric field. For example, when continued analytically to the real frequency axis from the upper half plane, $\mathcal{K}$ goes over into the retarded function K^R, for which we have in the coordinate representation the following equation (compare with [1]):

$$\left\{\frac{\partial}{\partial t} + \frac{\pi^2 r_s}{3}(T^2 - T_c^2) + D(\hat{p} - 2\frac{e}{c}A)^2\right\} K^R(t,r;t',r') = \tag{2}$$

$$= \delta(t - t')\,\delta(r - r'),$$

An equation for the advanced function K^A is obtained by making in (2) the substitution $\partial/\partial t \to -\partial/\partial t$.

For the actual continuation of the sum (1) to the axis of the physical frequencies, it is necessary to represent $\mathcal{K}$ in (1) in the form of a series in powers of the vector-potential $\vec{A}$ and to use the procedure of [1]. The corresponding relations of [1] can be extended to our case by replacing throughout tanh(z/2T) by coth(z/2T), taking the integrals in the sense of the principal value. Before going over to the final answers, we present one intermediate formula. Thus, upon continuation the sum in (1) goes over into

$$T\sum_\omega \mathcal{K}_{\omega,\,\omega-\omega_0} \to \frac{T}{2}(K^R_{\omega_0,\,0} + K^A_{0,\,-\omega_0}) - \frac{1}{4\pi i}\frac{T}{\pi}\int d\epsilon\, d\epsilon \times$$

$$\times K^R_{\epsilon,\epsilon'}\, D\left[\left(\hat{p} - \frac{2e}{c}A\right)^2 - \hat{p}^2\right]_{\omega'} K^A_{\epsilon'-\omega',\,\epsilon-\omega_0}\left[\frac{1}{\epsilon'} - \frac{1}{\epsilon'-\omega'}\right]. \tag{3}$$

(We take into account throughout the fact that the field frequencies are small.) Using (2) and (3), changing over from Fourier components to an explicit dependence of the time, we obtain after a number of transformations

$$\delta j(r,t) = \frac{8\pi N e r T}{m^2 p_0}\int_{-\infty}^{t}(\hat{k} - \frac{2e}{c}A(t))K^R(t,r;r,r')K^A(r,r';t,r)\,dr\,dr. \tag{4}$$

In the form, the formula is valid for the case when we take the field of the current into account in (2). This should not be done for a thin film. Changing over to the momentum representation, we obtain from (4) and (2)

$$\delta j = \frac{8\pi N e^2 r T}{m^2 p_0} E\int_0^\infty v\,dv\int dk\, e^{-2Dk^2 v}\exp\left\{-\frac{2}{3}v[\pi^2 r_s(T^2 - T_c^2) + \right. \tag{5}$$

$$\left. + e^2 D E^2 v^2]\right\}.$$

For a film, $dk = d^2k/(2\pi)^2 L$, where L is the film thickness. In a weak field and at $T > T_c$ we obtain the additional conductivity [3, 4]

$$\frac{\Delta\sigma}{\sigma} = \frac{9T}{2\pi^2 p_0^2 \ell L r_s(T^2 - T_c^2)} \equiv \lambda << 1\ . \tag{6}$$

In the general case when $T < T_c$:

$$\delta j = |\lambda| \sigma E \int_0^\infty dx \exp\{x - \frac{4}{9} (\sigma E / \pi^2 j_c)^2 x^3\}. \quad (7)$$

In a strong field

$$\delta j \approx |\lambda| \sigma E \frac{1}{3} \Gamma\left(\frac{1}{3}\right)\left(\frac{3\pi^2 j_c}{2\sigma E}\right)^{2/3}.$$

Of course, this result does not depend on the sign of $T - T_c$. To the contrary, in weak fields the use of the saddle-point method in (7) yields

$$\delta j = |\lambda| \sigma E \pi \left(\frac{\pi\sqrt{3} j_c}{2\sigma E}\right)^{1/2} \exp \frac{\pi^2 j_c}{\sqrt{3}\sigma E}. \quad (8)$$

Thus, the contribution from the fluctuation increases rapidly. Namely, $\delta j/j_0 \sim 1$ if $\sigma E \sim \pi^2 j_c/\sqrt{3} \ln(1/|\lambda|)$. If it is recognized that in real experiment the quantity $|\lambda|$ in (6) reaches values on the order of 10^{-2}, it becomes clear that the current-voltage characteristic of the film has a perfectly noticeable section corresponding to negative differential conductivity. At suitable potential differences between the ends of the film, generation of radiation should be observed, the analog of Josephson oscillations in a tunnel junction. It is possible that this is precisely the effect observed in [7]. It would be of interest to measure in experiment that part of the curve of Fig. 1, which co-responds to strong fields. Of course, (8) is valid only if $\delta j/j_0 \ll 1$. It is impossible to obtain theoretical formulas in the case when $\delta j \sim j_0$ (the fluctuations are nonlinear).

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STATIONARY STATE OF ELECTRONS IN A NON-EQULIBRIUM RADIATION FIELD

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We determine here the stationary state of a system of free nonrelativistic electrons situated in a non-equilibrium radiation field. It is assumed that the electrons scatter the radiation by the Compton mechanism. The role of bremsstrahlung processes in the field of the nuclei will be considered separately.