$$\delta j = |\lambda| \sigma E \int_0^\infty dx \exp\{x - \frac{4}{9}(\sigma E/\pi^2 j_c)^2 x^3\}. \qquad (7)$$

In a strong field

$$\delta j \approx |\lambda| \sigma E \frac{1}{3} \Gamma\left(\frac{1}{3}\right)\left(\frac{3\pi^2 j_c}{2\sigma E}\right)^{2/3}.$$

Of course, this result does not depend on the sign of $T - T_c$. To the contrary, in weak fields the use of the saddle-point method in (7) yields

$$\delta j = |\lambda| \sigma E \pi \left(\frac{\pi\sqrt{3} j_c}{2\sigma E}\right)^{1/2} \exp \frac{\pi^2 j_c}{\sqrt{3}\sigma E}. \qquad (8)$$

Thus, the contribution from the fluctuation increases rapidly. Namely, $\delta j/j_0 \sim 1$ if $\sigma E \sim \pi^2 j_c/\sqrt{3}\ \ln(1/|\lambda|)$. If it is recognized that in real experiment the quantity $|\lambda|$ in (6) reaches values on the order of 10^{-2}, it becomes clear that the current-voltage characteristic of the film has a perfectly noticeable section corresponding to negative differential conductivity. At suitable potential differences between the ends of the film, generation of radiation should be observed, the analog of Josephson oscillations in a tunnel junction. It is possible that this is precisely the effect observed in [7]. It would be of interest to measure in experiment that part of the curve of Fig. 1, which co-responds to strong fields. Of course, (8) is valid only if $\delta j/j_0 << 1$. It is impossible to obtain theoretical formulas in the case when $\delta j \sim j_0$ (the fluctuations are nonlinear).

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STATIONARY STATE OF ELECTRONS IN A NON-EQULIBRIUM RADIATION FIELD

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We determine here the stationary state of a system of free nonrelativistic electrons situated in a non-equilibrium radiation field. It is assumed that the electrons scatter the radiation by the Compton mechanism. The role of bremsstrahlung processes in the field of the nuclei will be considered separately.

Let us consider the case when the radiation field is isotropic[1)]. We assume that the radiation density is large and the electron density is small. Under these conditions, the electrons can be regarded as a heavy impurity and electron-electron and electron-nucleus collisions can be neglected (it will become clear later that these collisions do not influence the results). The radiation field, which is characterized by a photon distribution function N(k), is assumed to be stationary as a result of external factors.

It has turned out here that the stationary electron momentum distribution is Gaussian, i.e., it coincides with the Maxwellian energy distribution. The effective temperature of the electrons is expressed in terms of the emission spectrum. The prove the foregoing statements, let us examine the kinetic equation for the electron distribution function $f(\vec{p})$. The electron momentum changes in each individual interaction act by an amount $\Delta p \sim |\vec{p}\,' - \vec{p}| \sim k \ll p$, where k is the modulus of the photon wave vector. Consequently, the electron motion in momentum space is of the diffusion type and is described by the Fokker-Planck equation. It is easy to obtain with the aid of the usual procedure [1]:

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial p_i}\left[\frac{\langle \Delta p_i \Delta p_k \rangle}{2}\frac{\partial f}{\partial p_k} - \langle \Delta p_i \rangle f\right], \quad (1)$$

where in the first order in p/mc

$$\langle \Delta \mathbf{p} \rangle = \frac{\mathbf{p}}{p}\int \sigma c [1 + N(k')] N(k) (\Delta \mathbf{p})_{\parallel} \, d\mathbf{k}\, d\alpha, \quad (2)$$

$$\langle \Delta p^2 \rangle = \int \frac{\sigma c}{3} [1 + N(k)]\, N(k)\, (\Delta \mathbf{p})^2 \, d\mathbf{k}\, d\alpha, \quad (3)$$

$$\langle \Delta p_i \, \Delta p_k \rangle = 0, \quad i \neq k. \quad (4)$$

By $(\Delta\vec{p})_{\parallel}$ we denote here the projection of $\Delta\vec{p}$ on the only preferred direction, namely the electron momentum $\vec{p}$ in the laboratory frame, and $\sigma d\alpha$ is the differential Thompson scattering cross section. To calculate $\langle \Delta p^2 \rangle$ we can use the well known expression for $\Delta\vec{p}$ [2]. We then obtain directly

$$\langle \Delta p^2 \rangle = \frac{64\pi^2}{9}\left(\frac{e^2}{mc^2}\right)^2 \int [1 + N(k)] N(k) \hbar^2 c k^4 dk. \quad (5)$$

To find $\langle \Delta\vec{p} \rangle$ it is convenient to change over to a reference frame in which the electron is at rest. In this reference frame, the photon flight directions before and after scattering can be characterized by the angles θ, φ and θ', φ' respectively, between the preferred direction $\vec{p}$ and the vectors $\vec{k}$ and $\vec{k}\,'$. Then $(\Delta\vec{p})_{\parallel}$ takes the simple form

$$(\Delta p)_{\parallel} = k(\cos\theta - \cos\theta'). \quad (6)$$

Performing the appropriate calculations, we obtain

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1) The case of an anisotropic radiation field will be treated in our next communication.

$$\langle \Delta p \rangle = -4A \int N(k) k^3 dk, \tag{7}$$

where A is the corresponding angle integral. Substituting the values of the kinetic coefficients in (1), we see that its solution is a Maxwellian distribution with effective temperature

$$\theta = \frac{\int N(k)\,[1 + N(k)]\hbar c\, k^4 dk}{-A \int \frac{\partial N(k)}{\partial k} k^4 dk} = \frac{\int N[1 + N] k^4 dk}{B\epsilon} \tag{8}$$

$$\epsilon = \hbar c \pi^{-2} \int N k^3 dk .$$

The value of A is determined from the condition that in the case of a non-equilibrium photon distribution

$$N(k) = [\exp \frac{\hbar c k + \mu}{T} - 1]^{-1}$$

the condition $\theta = T$ must be satisfied. In this case $A = 1$ regardless of the value of μ.

It is easy to show that a Maxwellian distribution with an analogous expression for the temperature holds for an arbitrary effective scattering cross section.

The meaning of the result is that at small momentum transfer in the elementary interaction act, the electron motion in momentum space is Brownian. After a relaxation time $\tau = mc/\sigma\epsilon$, where ϵ is the radiation energy density, a random (Maxwellian) distribution with temperature θ is established.

Two remarks can be made with respect to the kinetic coefficients. The deceleration force (7) depends only on the total radiation energy density (if the scattering cross section is constant) and contains no effects connected with the Bose statistics of the photons. Moreover, it turns out that the diffusion coefficient, owing to the presence of the term N^2, also has a nonzero value in the classical limit $\hbar \to 0$.

A curious example are electrons in intergalactic space. The spectrum of the intergalactic radiation is a superposition of equilibrium Planck radiation with temperature 2.7°K and radiation of radio sources with a power-law spectrum $F(k) \sim k^{-\alpha}$, where $\alpha = 0.75$. Such a spectrum $F(k)$ corresponds to $N(k) \sim k^{-\alpha-3}$ and the integral in the numerator diverges when $k \to 0$.

The divergence of the contribution of the radio sources points to the need for a more detailed analysis.

The possibility of neglecting absorption and emission of quanta in the field of the nucleus depends not only on the gas density, but also on the frequency, since the bremsstrahlung cross section increases rapidly with decreasing frequency. In addition, F is a power-law spectrum only up to a certain frequency ω_{min}. Rough estimates show that the time of establishment of the stationary state of the electrons at the present time is larger than the cosmological one. In the past, however, when the electron density was approximately 10^3 times larger than the present one, their relaxation times equalled the cosmological time. If powerful sources of radio emission existed at that time, this could exert a strong influence on the electron temperature, leading in turn to a distortion of the relict radiation spectrum. The possibility of other astrophysical consequences is presently under study.

Let us stop to discuss briefly the history of the problem. It is well known that electrons and radiation with equal temperatures are in identical equilibrium with respect to all processes, particularly Compton scattering. Dreicer [1] wrote down a kinetic equation for the electrons and reduced it to the Fokker-Planck form. We were unable, however, to find a published solution for an arbitrary photon spectrum.

Knowing that the electron distribution is Maxwellian, we can determine the temperature from the condition that the derivative of the radiation energy density, calculated in accord with the Kompaneets formulas, must vanish.

In conclusion, we take the opportunity to thank R. A. Syunyaev for a discussion.

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ANTIANALOGS AND GAMMA DECAY OF ANALOG STATES

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When a proton is added to an even-even target nucleus, states with two isospin values may be produced:

$$\psi_{\nu;T_>,T_<} = \frac{1}{\sqrt{2T_0+1}}\phi_\nu(p)\,\psi_{0;T_0,T_0} + \sqrt{\frac{2T_0}{2T_0+1}}\phi_\nu(n)\,\psi_{0;T_0,T_0-1}, \tag{1}$$

$$\psi_{\nu;T_<,T_<} = \sqrt{\frac{2T_0}{2T_0+1}}\,\phi_\nu(p)\,\psi_{0;T_0,T_0} - \frac{1}{\sqrt{2T_0+1}}\phi_\nu(n)\,\psi_{0;T_0,T_0-1}, \tag{2}$$

where $\phi_\nu(p)$ and $\phi_\nu(n)$ are the wave functions of the proton and the neutron in the state ν; $\psi_{0;T_0,T_0}$ and $\psi_{0:T_0,T_0-1}$ are the wave functions of the ground state of the target and of its isobaric analog, $T_0 = (N - Z)/2$ is the isospin of the target, and $T = T_0 \pm 1/2$. The state (1) is the isobaric analog of the state

$$\psi_{\nu;T_>,T_>} = \phi_\nu(n)\,\psi_{0;T_0,T_0} \tag{3}$$

of a nucleus of the type of target plus neutron, and the state (2) is customarily called the antianalog [1].

It is usually assumed (cf., e.g., [2]) that the antianalog is distributed over a larger number of states, owing to the residual interaction. Indeed, the antianalog has the normal isospin of the given nucleus, and one might expect no hindrances whatever with respect to interactions that mix it with other states. However, the available data on the gamma decay of the analog states of nuclei of the sd shell [3] indicates that the mixing interaction in these nucei is weak for some reason. They show that in an appreciable number of the cases the