substances with close transitions. An analysis of the anomalies of the physical properties in such substances can be carried on a unified basis, in analogy with the foregoing one for Rochelle salt.

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SHOWER ENHANCEMENT OF BRANCH-POINT CONTRIBUTION AND INCREASE OF TOTAL CROSS SECTIONS AS A → ∞

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The values of the total $\pi^{\pm}N$ and $k^{-}N$ cross sections obtained [1] with the Serpukhov accelerator at 40 - 60 GeV turned out to be higher than the Regge (pole) curves or the curves of the "optical" or "eikonal" approximation [2]. The data [1] give the impression of a violation of the Pomeranchuk theorem, if it is assumed that $\sigma^{(tot)}$ takes on a constant asymptotic value $\sigma^{(tot)}$ = const already at E ≥ 20 GeV, since here $\sigma^{(tot)}_{k^-p}$ turns out to be larger by almost 3 mb than $\sigma^{(tot)}_{k^+p}$, which is almost constant still at E ≤ 20 GeV [3].

They can be understood, however, within the framework of the scheme of complex angular momenta [2, 4], if: (a) $\alpha_P'(0)$ and $\alpha_\omega(0)$ are shifted from the values ∿ 0.5 used earlier to the region ∿ 0.3; (b) the residue of the P pole, i.e., the value of $\sigma_\infty = \sigma^{(tot)}(E \to \infty)$ is greatly increased (by 10 - 15 mb); (c) besides the "optical" approximation diagrams b and c of Fig. 1, account is taken also of diagrams d and e, which correspond to shower production upon rescattering and greatly increase the contribution of the branch points.

All this leads to the appearance of a deep minimum [4] on the $\sigma^{(tot)} = \sigma(E)$ curves at E ∿ 40 - 60 GeV, in place of a minimum [2] at E ∿ 10^4 GeV in the "eikonal" model.

The most important is the first PP branch point (the branch points at the other singularities and the following P branch cuts are hardly noticeable). Taking only these into account, we obtain for the average of the π^-p and π^+p cross sections (or k^-p and k^+p, or $\bar{p}p$ and pn, or pp and pn):

$$\sigma^{(tot)} = \sigma_\infty [1 - \frac{\beta}{\xi + \xi_P} + \sum_{a + P; \omega} r_a e^{-(1-\alpha_a(0))\xi}], \quad (1)$$

where $\xi = \ln(E, \text{GeV})$ and $\xi_P = R_P^2/\alpha_P'(0)$ is known, since the radii R_P of the Regge P vertices are determined by the dependence of d /dt on t; for $\pi^{\pm}p$ [2] and $k^{\pm}p$ we have respectively $R_P^2 \approx 1.4$ and 1.5 $(GeV/c)^{-2}$. The parameters σ_∞, r_a, $\alpha_a(0)$ and β are chosen to suit, with $\beta = C\beta_0$, $\beta_0 = \sigma_\infty/32\pi\alpha_P'(0)$, and $\alpha_P'(0) = 0.47\ (GeV/c)^{-2} = 0.47 \times 0.39$ mb. In the "eikonal" model $\beta = \beta_0$, i.e., C = 1, and when account is taken of diagrams

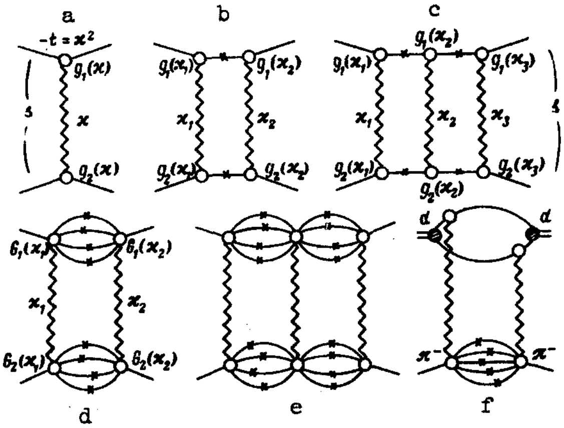

Fig. 1

[1)] Reported at Conference on High Energy Physics, Kiev, 22 October 1969.

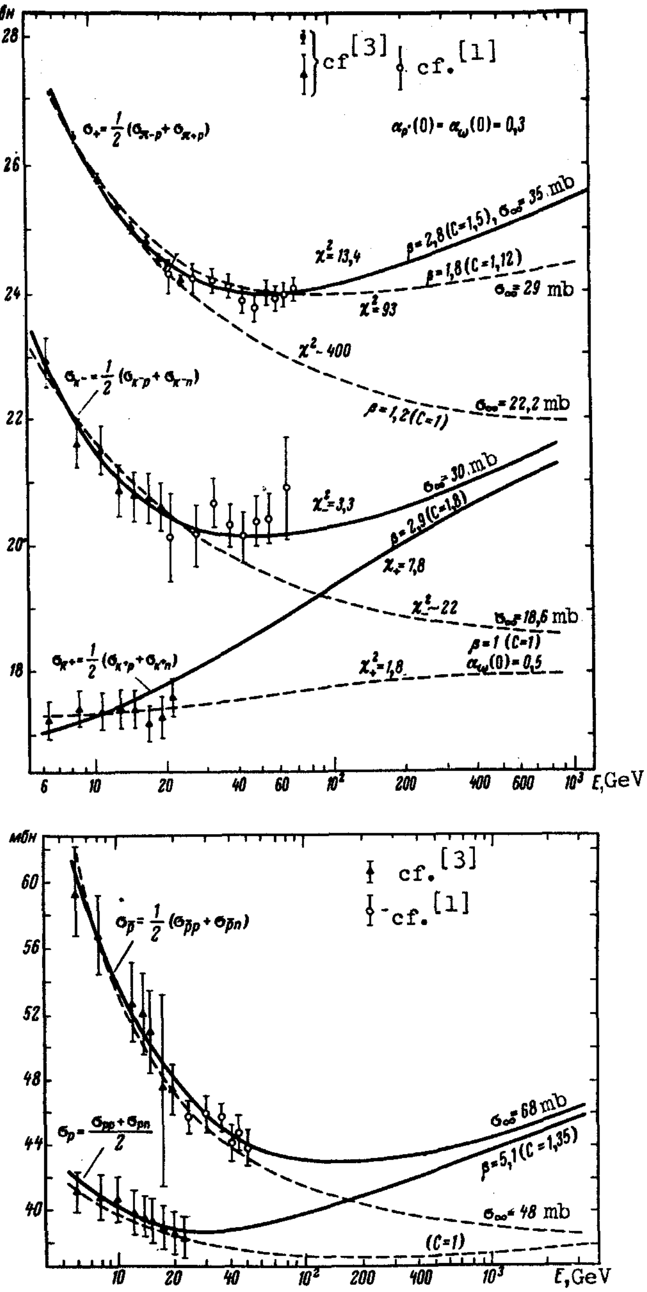

Fig. 2

d and e with "showers," the unitarity condition [4] yields C > 1.

The best values of the parameters σ_∞, $r_{P'}$, r_ω, $\alpha'_P(0)$, $\alpha_\omega(0)$ and β (i.e., C) are listed in the table and are plotted in Figs. 2a and b (solid curves). The "eikonal" curves (C = 1) are indicated by the dashed lines in Fig. 2; an additional curve is indicated for $\pi^\pm N$ with C = 1.12 intermediate between C = 1 and the optimal C = 1.5. It is seen that χ^2 increases sharply as C increases from C = 1.

It is seen from Fig. 2 and from the table that: (a) if the experimental data [1] are correct, then $\sigma^{(tot)}$ at $E \to \infty$ are tremendous, larger by 10 - 15 mb than in the case of E = 40 GeV; (b) the growth of $\sigma^{(tot)}$ with increasing E is, however, very slow, increasing only by 0.7 - 1 mb on going from E = 60 to E = 200 GeV; (c) the fastest to grow in the region E ∿ 60 - 1000 GeV are $\sigma^{(tot)}_{k^+N}$ and $\sigma^{(tot)}_{pN}$. If the foregoing scheme is correct, then $\sigma^{(tot)}_{k^+N}$ should increase by ∿ 1.5 mb on going from 10 to 60 GeV.

We note the following:

1. If only the PP branch points are taken into account, i.e., in the approximation in which (1) has been obtained, the absolute values obtained for the ratio $(\mathrm{Re}A/\mathrm{Im}A)_{t=0}$ are too large. Allowance for all branch points yields

$$\sigma^{(tot)} = \mathrm{Im}\, A(E, 0),$$

where

$$A = \sigma_\infty [f_P(\xi) + \sum_{\alpha = P', \omega} r_\alpha f_\alpha(\xi) e^{-(1-\alpha_\alpha(0))\xi}], \qquad (2)$$

$$f_P(\xi) = i[1 - \frac{\gamma_{PP}}{\lambda_P} \sum_{n=1}^{\infty} \frac{(-\gamma'_{PP}/\lambda_P)^{n-1}}{(n+1)(n+1)!}],\ f_\alpha(\xi) = r_\alpha \eta_\alpha [1 -$$

$$- \gamma_{\alpha P} \sum_{n=1}^{\infty} \frac{(-\gamma'_{\alpha P}/\lambda_P)^{n-1}}{n!(\lambda_P + n\lambda_\alpha)}],$$

$$\lambda_\alpha = R_\alpha^2 + \alpha'_\alpha(0)(\xi - i\pi/2),\ \alpha = P, P', \omega, \rho, A_2 \text{ и т.д.}$$

Particle	$\pi^{\pm}N$	$\pi^{\pm}N$[Eq.(2)]	$k^{\pm}N$	$NN-\bar{N}N$
$\sigma^{(tot)}$, mb	35.0	32.6	30.0	68.5
$r_{p'}$	1.1	1.26	0.75	0.95
r_{ω}	–	–	0.33	0.51
β	2.8	2.9	2.9	5.1
C_p	1.5	1.65	1.8	1.35
ξ_p	3.0	3.0	3.1	6.5
$a_{p'}(0)$	0.30	0.15	0.3	0.3
$a_{\omega}(0)$	–	–	0.3	0.3
χ^2	134 for 18 points	134 for 18 points	11.1 for 23 points	–

where $\gamma_{PP}/4\alpha'_P(0) = \beta$, and $\gamma'_{PP} \approx \gamma_{PP}$, γ_{aP}, and $\gamma'_{aP} \approx \gamma_{aP}$ are new parameters; η_a is the signature factor, and $\tau_a = \pm1$ is the signature, while

$$r_a \eta_a = i - r_a(\text{ctg}\frac{\pi a_a(0)}{2})^{r_a}.$$

When $\gamma_{PP} = \gamma'_{PP} \equiv \gamma_{aP} \equiv \gamma'_{aP} = 4\alpha'_P(0)\beta$, this formula yields a plot of $\sigma^{(tot)} = \sigma(E)$ which is almost indistinguishable from (1), but at somewhat different parameters σ_∞, r_a, and β (see the table). At the same time, it yields very good values for $(\text{ReA}/\text{ImA})_{t=0}$ (unlike the case when only PP branch points are taken into account).

2. Of importance for the verification of the indicated scheme are data [5] on the dependence of the cross section $d\sigma_{reg}/dt$ of the regeneration $k_{20} \to k_{10}$ on nucleons on the energy E, and on the phase of its amplitude. If $\alpha_\omega(0) = 0.3$ (but not close to $j = 1$), then $d\sigma_{reg}/dt$ should decrease like $E^{-2(1-\alpha_\omega(0))} = E^{-1.4}$, and its value at $t = 0$

$$(d\sigma_{reg}/dt)_{t=0} = \frac{(\Delta\sigma)^2}{16\pi}\{1 + (\text{Re}\,A_\omega/\text{Im}\,A_\omega)^2\}$$

should not exceed noticeably the optical limit $(\Delta\sigma)^2/16\pi \approx 0.4$ mb$(\text{GeV}/c)^{-2}$, where $\Delta\sigma = \sigma^{(tot)}_{k^-p} - \sigma^{(tot)}_{k^+p} \approx 3$ mb.

3. The difference $\Delta_\pi\sigma$ between the π^-p and π^+p cross sections is described by

$$\Delta_\pi\sigma = \sigma_p\,\text{Im}\,f_p(\xi)\,e^{-(1-a_p(0))\xi}$$

with $\alpha_p(0) = 0.45$; however, in the region of $E \sim 60$ GeV, it is smaller than the value given in [1] - at the limit and beyond the limit of errors. This may be due to the fact that no account was taken in [1] of the shower formation effect in the Glauber correction $\Delta_G\sigma$ as a result of the diagram of Fig. 1c. Allowance for this effect increases $\Delta_G\sigma$ (by about 0.5 - 1 mb); this increase is the larger, the larger E, i.e., it raises

$$\sigma^{(tot)}_{\pi^+p} = \sigma^{(tot)}_{\pi^-d} - \sigma^{(tot)}_{\pi^-p} + \Delta_G\sigma$$

and accelerates the decrease of the experimental values of $\Delta_\pi\sigma$ with increasing E.

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REGENERATION OF K^0 MESONS AND THE POMERANCHUK THEOREM[1)]

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1. According to data obtained recently in Serpukhov [1], the total cross sections for K^- mesons are constant in the interval 20 - 55 GeV and amount to

$$\sigma_{K^-p} = 21 \pm (0.3 - 0.5)\ \text{mb}, \quad \sigma_{K^-d} = 40 \pm (0.4 - 0.8)\ \text{mb},$$
$$\sigma_{K^-n} = 20 \pm (0.5 - 1.3)\ \text{mb}.$$

The previously obtained [2] data on K^+ mesons are as follows: the cross section σ_{K^+p}, neglecting small deviations (~0.2 - 0.3 mb, but at an error of 0.1 mb) is constant in the 6 - 20 GeV/c interval and equals 17.3 mb; $\sigma_{K^+d} = 33.8 \pm 0.3$ mb and $\sigma_{K^+n} = 17.5 \pm 0.4$ mb. (There are no data on K^+ mesons at higher energies). Thus, the data of [1,2] do not contradict the hypothesis that the cross sections of K^+ reach the constant asymptotic limit already starting with 6 GeV/c, and the corresponding value for K^- is 20 GeV/c,[2)] so that as $E \to \infty$ we have

$$\sigma_{K^-p} - \sigma_{K^+p} = 3.5 \pm 0.5\ \text{mb}, \quad \sigma_{K^-d} - \sigma_{K^+d} = (6 \pm 1)\ \text{mb}$$
$$\sigma_{K^-n} - \sigma_{K^+n} = 2.5 \pm 1.5\ \text{mb}. \tag{1}$$

Were it not so, this would mean violation of the Pomeranchuk theorem [3], according to which $\sigma = \bar{\sigma}$ as $E \to \infty$.

Of course, the data of [1, 2] do not prove in any manner that the cross sections σ_{K^+p} and σ_{K^-p} have actually reached their asymptotic constant limits. In particular, one cannot exclude the possibility that the cross section σ_{K^+p} begins to grow, starting with 20 GeV, and to approach σ_{K^-p}. As to σ_{K^-p}, it can be approximated near 20 GeV, at the present accuracy, not only by a horizontal line, but also by a curve having a minimum (the latter possibility

1) Reported at conference on High Energy Physics, Kiev, 22 October 1969.

2) The data of [2] do not contradict the assumption that σ_{K^-p} and σ_{K^-d} reach the constant limits at 18 and 14 GeV/c, respectively.