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SUSCEPTIBILITY AND HEAT CAPACITY OF A METAL PLATE

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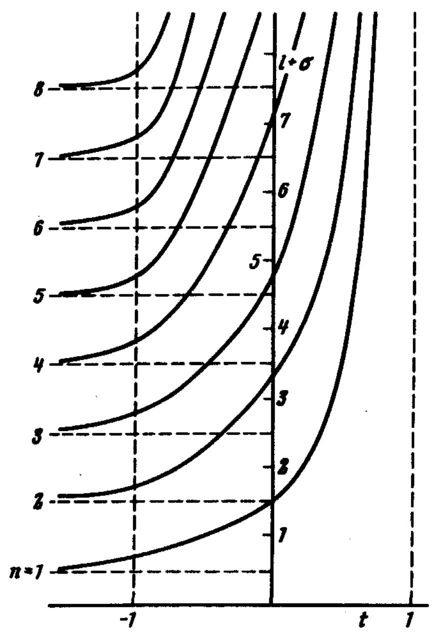
It is well known at present that in a magnetic field H parallel to the surface of a metal there exist, besides the volume Landau levels, also surface magnetic levels (SML). They were first observed by Khaikin [1], who investigated the oscillations of surface impedance. The nature of these oscillations remained unexplained until the publication of the paper by Nee and Prange [2], in spite of the fact that I. Lifshitz and Kosevich [3] investigated the quantization due to the presence of the metal boundary even earlier, in connection with the de Haas - van Alphen effect.

The present paper is devoted to the contribution of SML to the thermodynamic properties. For simplicity, we consider a metal occupying the half-space $x > 0$, choose the z axis along the magnetic field, and assume the electron spectrum to be quadratic and isotropic. The SML are characterized by the following quantum numbers: the number $n = 1, 2, \ldots$, the tangential projection of the electron momentum $\{p_y, p_z\}$, and the spin quantum number $\sigma = 1/2$. The dependence of $\varepsilon_{n\sigma}(p_y, p_z)$ on p_y is shown in the figure. When $p_y < -(2m\varepsilon^{(s)} - p_z^2)^{1/2}$, the distance from the surface to the center of the classical orbit exceeds the Larmor radius R, the electron does not collide with the surface, and the SML have an exponentially small deviation (with respect to p_y) from the Landau levels. The region $|p_y| < (2m\varepsilon^{(s)} - p_z^2)^{1/2}$ corresponds to orbited that intersect the surface, and when $p_y > (2m\varepsilon^{(s)} - p_z^2)^{1/2}$ the entire orbit is located outside the metal and there are no SML. A feature of the spectrum is that all the levels with numbers $n < \varepsilon_F/\hbar\Omega$, where $\Omega = eH/mc$ (the electron charge is e) intersect the Fermi level ε_F. Because of this, the contribution $M^{(s)}$ of the SML to the magnetic moment exceeds, under certain conditions, the usual magnetism $M^{(v)}$ of the conduction



Spectrum of surface magnetic levels $\epsilon^{(s)}_{n\sigma}(p_y, p_z)$. $l = (\epsilon^{(s)} - p_z^2/2m)/\hbar\Omega$, $t = p_y(2m\epsilon^{(s)} - p_z^2)^{1/2}$, $\sigma = \pm 1/2$

electrons.

We begin the calculation with the density of states $dz/d\varepsilon$. It is shown in [4] that the density of states can be broken up into two parts, one proportional to the volume and connected with the Landau levels, and the other proportional to the surface area S and equal to the SML density

$$dz/d\epsilon = \frac{S}{(2\pi\hbar)^2}\left\{\sum_{n\sigma}\int dp_y\,dp_z\left[\delta(\epsilon-\epsilon^{(s)}) - \frac{1}{2}\delta(\epsilon-\epsilon^{(v)})\right] - \right. \tag{1}$$

$$\left. - \frac{1}{\pi}\operatorname{Im}\sum_{\sigma}\int dp_y\,dp_z\,\frac{\partial}{\partial\epsilon}\ln D_{\ell-\frac{1}{2}+\sigma}(\zeta)\right\},$$

where

$$\zeta = 2p_y(2m\hbar\Omega)^{-1/2},\quad \ell = \left(\epsilon - \frac{p_z^2}{2m}\right)/\hbar\Omega,\quad \epsilon^{(v)} = \frac{p_z^2}{2m} + \hbar\Omega\left(n - \frac{1}{2} + \sigma\right)$$

The SML are determined by the solution of the equations

$$D_{\ell-\frac{1}{2}+\sigma}(\zeta) = 0 \tag{2}$$

D is the parabolic-cylinder function. We denote these solutions by $\ln\sigma(\zeta)$, change over in (1) to the variables ℓ and ζ, and integrate with respect to ℓ with the aid of δ-functions and with respect to ζ by parts. We then obtain the following result for the number of states with energy lower than ε:

$$Z(\epsilon) = \frac{Sm\Omega}{2\pi^2\hbar}\left\{\sum_{n\sigma}\int_{n-\frac{1}{2}+\sigma}^{\epsilon} t_{n\sigma}(\ell)\left(\frac{\ell}{\epsilon-\ell}\right)^{1/2}d\ell - \frac{1}{\pi}\operatorname{Im}\sum_{\sigma}\int d\ell\,d\zeta\sqrt{\epsilon-\ell}\,\frac{\partial}{\partial\ell}\ln D\right\}, \tag{3}$$

where $\varepsilon = \varepsilon/\hbar\Omega$, $t_{n\sigma}(\ell)$ is the solution of Eq. (2), in which $\zeta = 2t\ell^{1/2}$, and the summation is over those n for which the lower limit in the integral is smaller than the upper limit. If $\varepsilon\ \varepsilon_F \gg \hbar\Omega$, then the principal role in (3) is played by the first expression, and it is necessary to take into account many terms in the sum over n, so that a quasiclassical representation of the SML spectrum can be used. We present an expression for the magnetic-field-dependent increment to the number of states

$$Z(\epsilon) = \frac{Sm\Omega}{2\pi^3\hbar}\sum_{\nu=-\infty}^{\infty}{}'(i\nu)^{-1}\int_0^{\epsilon}\left(\frac{\ell}{\epsilon-\ell}\right)^{1/2}d\ell\int_{-1}^{1}dt\,\exp\left\{2\pi i\nu\left[\ell\mu(t) + \frac{1}{4}\right]\right\}, \tag{4}$$

where

$$\mu(t) = \frac{2}{\pi}\int_t^1 dt_1\,(1 - t_1^2)^{1/2}$$

there is no term with $\nu = 0$, as is designated by the primed summation sign. When $\varepsilon \gg 1$, the integral depends mainly on the region $1 - t \ll 1$ and on the values $\ell \sim \varepsilon$.

Omitting the oscillating terms (which is of no interest because it has the same period in H^{-1} as the de Haas - van Alphen term in the density of the volume states), we get

$$Z(\epsilon) = \frac{Sm\Omega}{\hbar}\left(\frac{\epsilon}{\hbar\Omega}\right)^{1/3}A;$$

$$A = \frac{3^{1/6}\,\Gamma(5/6)}{2^{1/3}\pi^{3/2}\Gamma^2(1/3)}\sum_{\nu=1}^{\infty}\nu^{-5/3}\sin\pi\left(\frac{\nu}{2}+\frac{1}{3}\right) = 0{,}03\cdot 10^{-2}. \tag{5}$$

With the aid of (5) we obtain the contribution of the SML to the magnetic moment

$$M^{(s)} = -\frac{e\hbar}{2mc}\,\frac{Sm}{\hbar^2}\,\frac{\epsilon_F^{4/3}}{(\hbar\Omega)^{1/3}}\,A \tag{6}$$

and the field-dependent heat capacity

$$C_H^{(s)} = T\,\frac{Sm}{\hbar^2}\left(\frac{\hbar\Omega}{\epsilon_F}\right)^{1/3}\frac{\pi^2}{9}\,A\,. \tag{7}$$

Formula (6) coincides, apart from a coefficient and the sign (paramagnetism), with the results of Steele [5] and Dingle [6], obtained in the quasiclassical approximation even prior to the work on the SML [2]. Doubts concerning the validity of [5] and [6] were expressed in [7] and [8]. In [7], the influence of the magnetic field on the spectrum was taken into account by perturbation theory, and the singularity was lost (near the end point of the spectrum, i.e., at $t = 1$):

$$\epsilon_{n\sigma}^{(s)} = (p_y^2 + p_z^2)2m + \left[\frac{3\pi}{2}\left(n - \frac{1}{4}\right)\hbar\Omega\right]^{2/3}\left(\frac{p_y^2}{2m}\right)^{1/3} + \hbar\Omega\sigma. \tag{8}$$

In [8], the metal boundary was simulated by a parabolic potential of certain frequency ω_0, the smallness of the field H was understood in the sense of $\Omega << \omega_0$, and the usual Landau expression was obtained for the susceptibility.

Let us discuss the influence of: 1) the finite dimension of the metal in the x direction denoting the corresponding dimension by L_x: 2) the non-ideal character of the surface, and 3) the collisions of the electrons with the volume defects (impurities)[1].

1. Expression (5) is determined by the glancing electrons (relative to the surface), whose wavelength in the x direction is

$$\lambda_x \quad \left(\frac{\epsilon_F}{\hbar\Omega}\right)^{1/3}\frac{\hbar}{p_F}$$

(see [8]). In order for (6) and (7) to be valid we must have $L_x >> \lambda_x$. We emphasize that L_x must be compared with λ_x, and not with R, as was done in [7] and [8]. The foregoing inequality limits $M^{(s)}$ when $H \to 0$. However, the maximum value of $M^{(s)}/M^{(x)}$ with respect to L_x turns out to be larger

$$(M^{(s)}/M^{(v)})_{max} \quad \left(\frac{\epsilon_F}{\hbar\Omega}\right)^{4/3}\left(\frac{\hbar}{L_x p_F}\right)_{max} \quad \frac{\epsilon_F}{\hbar\Omega}.$$

2. Surface roughness leads to damping of the MSL. Depending on the ratio between the size of the roughness a, the dimension of the smooth sections of the surface d, and the wavelenth λ_x, the requirement that the surface have good specular properties takes on different forms [4]:

$$(ap_F)^2\Omega/\epsilon_F(\hbar p_F d)^{1/2} << 1 \qquad \text{if} \quad \left(\frac{\hbar\Omega}{\epsilon_F}\right)^{2/3} p_F d/\hbar << 1,$$

$$(ap_F)^2\left(\frac{\Omega}{\epsilon_F}\right)^{4/3}/\hbar^{2/3} << 1 \qquad \text{if} \quad \left(\frac{\hbar\Omega}{\epsilon_F}\right)^{2/3} p_F d/\hbar >> 1.$$

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[1] One more limitation is connected with $dM/dB = 4\pi^{-1}$. This remark is due to I. A. Privorotskii [9].

3. Collisions with impurities do not change (6) or (7) if $\lambda_x << \ell_x$, where $\ell_x = \tau v_x$ is the mean free path in the x direction, i.e., if

$$(\hbar\Omega/\epsilon_F)^{2/3}\epsilon_F >> \hbar/\tau. \tag{9}$$

The meaning of condition (9) is that the distance between the MSL should be large compared with the damping h/τ. The condition (9), which is the most stringent of all the foregoing, is nevertheless satisfied for fields 0.1 -10 Oe (it is meaningful to measure $M^{(s)}$ in this interval at a range ℓ_0 = 1 - 0.1 mm).

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FILM SUPERCONDUCTIVITY STIMULATED BY A HIGH-FREQUENCY FIELD

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A number of problems pertaining to nonlinear electrodynamics of small-size superconductors have been considered recently [1, 2]. In each case the result was that an alternating electromagnetic field, by decreasing the ordering parameter Δ, exerted a destructive action on the superconducting properties, just as in the case of constant magnetic field or current.

Yet, as will be shown here, such a situation obtains only for field frquencies below a certain critical value ω_c, and when $\omega > \omega_c$ the high-frequency field should increase Δ.

To illustrate the physical nature of this phenomenon, let us turn to the principal equation of the BCS theory [3]

$$\Delta = g\int_{\Delta}^{\hbar\omega_D} d\epsilon \frac{\Delta}{\sqrt{\epsilon^2 - \Delta^2}}[1 - 2n(\epsilon)], \tag{1}$$

which determines the dependence of the energy gap Δ on the distribution function $n(\varepsilon)$. At equilibrium we have $n(\varepsilon) = (e^{\varepsilon/T} + 1)^{-1}$. An alternating field of frequency $\omega < 2\Delta/h$, when absorbed by the excitations, will shift the "center of gravity" of $n(\varepsilon)$ towards larger values of ε, keeping the total number of excitations constant. Such a shift of n, as seen from (1), will lead to an increase of Δ, owing to the decrease of the level density with increasing distance from the threshold. The actual change of $n(\varepsilon)$ will be proportional to the field intensity E^2 (at not too large E) and to the energy relaxation time τ_0 of the excitations.