the radiation losses prevail, S_t = const, and $P_t \sim R^2$.

Thus, to maintain the plasma continuously in atmospheric air by means of a CO_2 laser, the required power is about 7 kW. This is an imposing figure, but not unrealistic. The discharge can be ignited practically anywhere, using mirrors and lenses to feed the energy. It is also easy to localize (to "stabilize the flame") by focusing the beam. At high pressures, the power required is much lower (it can be varied also by choosing different gases), but the construction of the "plasmotron" is in this case obviously more complicated. The complete solution of the problem of subsonic propagation of a light discharge will be published in Zh. Eksp. Teor. Fiz. (Sov. Phys.-JETP).

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OSCILLATORY APPROACH TO SINGULAR POINT IN THE OPEN COSMOLOGICAL MODEL

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It was shown in earlier communications [1, 2] that in the general cosmological solution of Einstein's equation there exists a singularity having a complicated oscillatory character. There was also considered a particular example of a homogeneous closed model (a world with homogeneous space of type IX after Bianchi), admitting of a more complete analysis [2] (this model was considered also by Misner [3]). We wish to report in this paper another similar example, which not only confirms once more the qualitative analysis of the general case, but casts additional light on certain aspects of the problem. We refer here to a model with homogeneous space of type VIII after Bianchi.

Let again $\vec{l}$, $\vec{m}$, and $\vec{n}$ be reference vector defining the coordinate dependence of the space metric, and let a(t), b(t), and c(t) be functions of the synchronous world time t, defining the scales of the spatial distances in the directions $\vec{l}$, $\vec{m}$, and $\vec{n}$, respectively. Homogeneous spaces of types VIII and IX correspond to constant values (independent of the coordinates) of the quantities

$$\lambda = (\vec{l}\cdot\mathrm{curl}\,\vec{l})/v, \quad \mu = (\vec{m}\cdot\mathrm{curl}\,\vec{m})/v, \quad \nu = (\vec{n}\ \mathrm{curl}\,\vec{n})/v, \quad v = \vec{l}\cdot\vec{m}\times\vec{n},$$

(the products $\vec{m}\cdot\mathrm{curl}\,\vec{l}$, $\vec{n}\cdot\mathrm{curl}\,\vec{l}$, etc. vanish). If the signs of these constants are all the same we have space of type IX, and if the sign of one of them is the opposite of the other two we have space of type VIII. In the former case we can put $\lambda = \mu = \nu = 1$, and in the latter case let $\lambda = -1$, $\mu = \nu = 1$.

The functions a, b, and c satisfy the Einstein equations

$$\alpha_{\tau\tau} = (\mu b^2 - \nu c^2)^2 - \lambda^2 a^4, \quad \beta_{\tau\tau} = (\lambda a^2 - \nu c^2) - \mu^2 b^4, \tag{1}$$

$$\gamma_{\tau\tau} = (\lambda a^2 - \mu b^2)^2 - \nu^2 c^4,$$

$$\alpha_\tau \beta_\tau + \alpha_\tau \gamma_\tau + \beta_\tau \gamma_\tau = \frac{1}{2}(\alpha + \beta + \gamma)_{\tau\tau}, \tag{2}$$

where $a = e^\alpha$, $b = e^\beta$, $c = e^\gamma$, and the variable τ is connected with t by the relation $d\tau = dt/abc$ (see [2], Sec. 4). The character of the replacement of the Kasner regimes in short epochs (using the terminology introduced in [1]) certainly does not depend on the signs of λ, μ, and ν, since it is determined each time by only one term in the right sides of (1), containing λ^2, μ^2, or ν^2. It is therefore necessary to consider only solutions describing "long epochs," in the course of which two of the functions a, b, and c experience multiple oscillations, while the third decreases monotonically (as $t \to 0$) and can be neglected in comparison with the other two. If the monotonically decreasing function is a, then the equations obtained when this function is neglected have the same form as in the analogous case for type-IX space; accordingly, the temporal evolution of the metric during the long epoch is described by identical formulas in both models.

On the other hand, if the function b or c decreases monotonically (say, c), then we obtain from (1) and (2), after neglecting this function (cf. [2]),

$$q_{\xi\xi} + \frac{1}{\xi} q_\xi + \operatorname{sh} q = 0, \qquad (\alpha + \beta)_{\xi\xi} = 0, \tag{3}$$

$$\gamma_\xi = -\frac{1}{2\xi} + \frac{\xi}{8}(q_\xi^2 + 2\operatorname{ch} q + 2), \tag{4}$$

where $q = \alpha - \beta$, and ξ is a variable connected with the variable τ by

$$\xi = \xi_0 \exp\left\{\frac{2a_0^2}{\xi_0}(\tau - \tau_0)\right\};$$

during the course of the long epoch, this variable runs through values that start with some initial very large $\xi \sim \xi_0$ down to $\xi \sim 1$. Equations (3) coincide with those for the type-IX model, while Eq. (4) differs in the sign of the last number 2 in the parentheses. As a result we obtain for the functions $a(\xi)$ and $b(\xi)$ the same equations as before, namely (in the first approximation in $1/\xi$)

$$\left.\begin{matrix} a \\ b \end{matrix}\right\} = a_0 \sqrt{\frac{\xi}{\xi_0}}\left[1 \pm \frac{A}{\sqrt{\xi}} \sin(\xi - \xi_0)\right] \tag{5}$$

(A is a constant), and for $c(\xi)$ and $t(\xi)$ we obtain

$$\frac{c}{c_0} = \frac{t}{t_0} = \exp\{-\frac{1}{8}(\xi_0^2 - \xi^2)\}$$ (6)

in place of the previous

$$\frac{c}{c_0} = \frac{t}{t_0} = \exp\{-A^2(\xi_0 - \xi)\}.$$ (6a)

Thus, the difference in the character of the long epochs in both cases reduces only to a different connection between the time t and the variable ξ, governing the oscillations of the functions (5). If t_0 and t_1 are the upper and lower time limits of the long epoch, then $8\ln(t_0/t_1) \approx \xi_0^2$ in the case of (6) and $A^{-2}\ln(t_0/t_1) \approx \xi_0$ in the case of (6a). On the other hand, the value of ξ_0 determines the total number of oscillations during the long epoch (equal to $\xi_0/2\pi$). It is clear therefore that at a specified ratio t_0/t_1 the number of oscillations is in general smaller in the case of (6) than in the case of (6a).

In connection with the foregoing, we can make the following two remarks.

1. The distinguishing feature of the type-IX model, compared with type VIII, is that when $\lambda = \mu = 1$ the difference $\lambda a^2 - \mu b^2$ in Eqs. (1) is small together with the difference a - b; such a contraction requires not only that the signs of λ and μ be identical, but also that these quantities be essentially constant. One can therefore expect, in the most general case of an inhomogeneous space metric, the character of its time dependence during long epochs will correspond to (6) and not to (6a). This conclusion is indeed confirmed by an analytic construction of a general solution for a long epoch, as will be shown elsewhere by V. A. Belinskii and I. M. Khalatnikov.

2. A homogeneous space of type VIII has an infinite volume, whereas a type-IX space is closed. Therefore the aggregate of these two examples is evidence of the absence of a direct connection between the oscillatory approach to the singular point and the openness or closedness of the model.

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STRUCTURE OF TURBULENT-VISCOSITY COEFFICIENT FOR AN ISOTROPIC TURBULENCE

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An isotropic-turbulence equation, containing turbulent viscosity, was proposed in [1]. By introducing the turbulent viscosity, various hypotheses regarding the closing of the system of equations of isotropic turbulence are replaced by some hypotheses concerning the structure of the coefficient of turbulent viscosity, defined by

$$\nu_T = k(r^2/t).$$ (1)

The quantity k may depend in general on the Reynolds number of the turbulence. This hypothesis