DIFFUSE BOUNDARY CONDITION FOR CONDUCTION ELECTRONS

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An important role is played in the theory of the anomalous effect [1] by the boundary condition that must be satisfied on the metal boundary by the distribution function

$$f(p_x, p) = (1 - \rho)f(-p_x, p) + \rho f_0(\epsilon). \tag{1}$$

Here $f(\pm p_x, p)$ is the distribution function for electrons traveling from the surface and in the opposite direction, respectively, f_0 is the equilibrium function, ρ is the so-called diffuseness coefficient, and $\vec{p} = \{p_y, p_z\}$ is the electron-momentum component tangent to the surface.

The phenomenological boundary condition (1) was improved by various means: ρ was assumed to be dependent on the angle of incidence, the last term in (1) was chosen to satisfy the condition that no current must flow through the surface, etc. The analysis, however, remained either phenomenological [2] or too general [3].

The present paper is devoted to a microscopic derivation of a boundary condition that holds for sufficiently low temperatures, when "thermalization" of the electrons by emission and absorption of phonons is impossible (the "thermalization" is described in (1) by f_0). The diffuseness of the reflection is regarded as a consequence of random roughness of the surface, and the statistical averaging is carried out in this sense. With respect to the magnitude of the roughness, the following is assumed (Fig. 1): Let a be the average size of the roughness and d the average dimension of the flat sections of the surface; generally speaking, these are quantities of atomic scale. It is assumed here that $(ap_x)^2/(pd)^{1/2} \ll 1$ for all the essential electrons, i.e., for all those contributing to the current (cf. the discussion of the results).

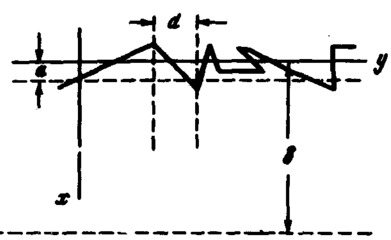

Fig. 1

Deferring the details of the derivation to a detailed article, we present here the result [1]:

$$f(p_x, p) = f(-p_x, p)\left[1 - p_x \int \frac{d^2p'}{\pi^2} p'_x \xi_2(p - p')\right] + \\ + p_x \int \frac{d^2p'}{\pi^2} p'_x \xi_2(p - p') f(-p'_x, p), \tag{2}$$

where $p_x = (2m\epsilon - p^2)^{1/2}$, $p'_x = (2m\epsilon - p'^2)^{1/2}$, $\xi_2(p)$ is the Fourier component of the binary correlation function $\xi_2(s - s') = \langle \xi(s)\xi(s') \rangle$, $\xi(s) = x$ is the equation of the rough surface, and for simplicity the electron spectrum is assumed to be quadratic and iso-

1) We put $\hbar = e = c = 1$ in the intermediate expressions.

tropic. The function $\xi_2(p)$ is a characteristic of the surface. The dimension of the region where it differs from zero is d^{-1}, its magnitude here is of the order of a^2d^2; ξ_2 determines the impedance, the damping of the surface levels discovered by Khaikin (this question is considered in [4]), the light reflection coefficient, etc.

It can be readily seen that condition (2) leads automatically to the vanishing of the electron flux through the surface in any energy interval $\Delta\epsilon$, and is identically satisfied for a function $f(\epsilon)$ that depends only on the energy. The factor p_x ensures almost-specular reflection for small incidence angles. The specularity improves in the limit $p_0 d \gg 1$ (p_0 is the Fermi momentum), since the terms with ξ_2 cancel each other in this case.

Let us consider the problem of the anomalous skin effect. Solution of the equation

$$v_x \frac{df}{dx} + \nu(f - f_0) = - vE(x) \frac{df_0}{d\epsilon} \qquad (\nu = \tau^{-1} + i\omega = v_0/\ell,$$

(ℓ is the effective mean free path) with boundary condition (2) makes it possible to find the connection between the current $j(x)$ and the field $E(x)$. Following a Fourier expansion with respect to the coordinate (we use an even continuation into the region $x < 0$), this connection takes the form

$$j(k) = \sigma(k)\mathcal{E}(k) + \int \frac{dk'}{2\pi}\, \sigma(kk')\mathcal{E}(k'), \tag{3}$$

where

$$\sigma(kk') = \frac{m^2\nu^2}{2\pi^5}\int d^2p\, d^2p'\, \frac{p_x p_x' p_y \xi_2(p-p')}{(kp_x)^2 + (m\nu)^2}\left[\frac{p_y'}{(k'p_x')^2 + (m\nu)^2} - \frac{p_y}{(k'p_x)^2 + (m\nu)^2}\right], \tag{4}$$

$\sigma(k)$ is the electric conductivity in specular reflection [1], the field is assumed directed along the y axis, and the integration is carried out over p and $p' < p_0$.

The asymptotic forms of $\sigma(kk')$ for $k \sim k' \gg \ell^{-1}$ and $p_0 d \gg 1$ are

$$\begin{aligned} \sigma(kk') &\sim -(ap_0^2)^2/(p_0 d)^{1/2}\ell kk'(k+k'), \quad |k\ell| \gg (p_0 d)^{1/2}\ln p_0 d \\ \sigma(kk) &\sim -(a p_0 \ell/d)^2 \ln|p_0 d/k^2\ell^2|, \quad |k\ell| \ll (p_0 d)^{1/2}\ln p_0 d' \end{aligned} \tag{5}$$

The field $\mathcal{E}(k)$ is obtained with the aid of (3) and Maxwell's equation

$$k^2\mathcal{E}(k) + 2E'(0) = 4\pi i\omega j(k). \tag{6}$$

We measure experimentally the impedance

$$Z = 4\pi i\omega E(0)/E'(0). \tag{7}$$

This formula enables us to determine the impedance from the known solution of Eqs. (3) and (6). However, with approximately the same accuracy as used in condition (2), we can expand Z in powers of $\sigma(kk')$ [2]. The obtained increment ΔZ of the impedance Z_0 calculated using the specular boundary condition depends on the ratio of the effective mean free path ℓ to the skin-layer depth δ, and also on the quantity $p_0 d$.

In the anomalous skin effect [3]

$$\left(|\delta/\ell| << 1;\ \delta = \left(\frac{c^2\hbar^3}{ie^2p_0^2\omega}\right)^{1/3};\ Z_0 \sim \omega\delta\right)$$

we have

$$\frac{\Delta Z}{Z_0} \sim \begin{cases} (p_0^2 a d)^2 \delta/\ell, & p_0 d << 1 \quad (8.1) \\ \dfrac{(a p_0)^2}{(p_0 d)^{1/2}} \dfrac{\delta}{\ell}, & 1 << (p_0 d)^{1/2} << |\ell| / |\delta| \ln p_0 d \quad (8.2) \\ \left(\dfrac{\ell a}{\delta d}\right)^2 \ln|\delta^2 p_0 d/\ell^2|, & |\ell/\delta| << (p_0 d)^{1/2} \ln p_0 d \quad (8.3) \end{cases}$$

In the normal skin effect [3]

$$\left(|\delta/\ell| >> 1;\ \delta = \left(\frac{3\pi c^2\hbar^3}{4e^2p_0^3\omega\ell}\right)^{1/2};\ Z_0 \sim \omega\tilde{\delta}\right)$$

we have

$$\frac{\Delta Z}{\tilde{Z}_0} \sim \begin{cases} (p_0^2 a d)^2 \ell/\tilde{\delta}, & p_0 d << 1 \quad (9.1) \\ \dfrac{a^2\ell}{d^2\delta} \ln p_0 d, & p_0 d >> 1 \quad (9.2) \end{cases}$$

A plot of $\Delta Z/Z_0$ against ℓ/δ (i.e., against the frequency ω when $\omega\tau >> 1$) is shown in Fig. 2.

In the region $|\ell| < |\delta|$, the main contribution is determined by the electrons that do not collide with the surface, and the boundary condition is obviously not needed for the calculation of $\tilde{Z}_0$. The increment ΔZ is connected with the electrons located at a distance smaller than $|\ell|$ from the surface and moving mainly along the surface, i.e., with $p_x < p_0$. There is no strong inequality here, making it possible to calculate the coefficient in (9) only if $(ap_0)^2/(p_0 d)^{1/2} << 1$.

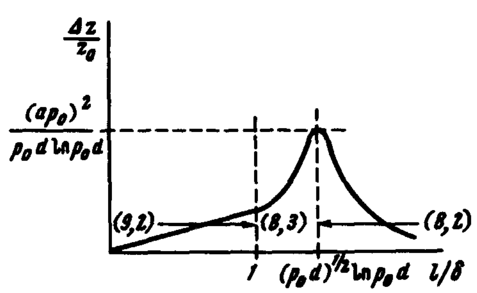

Fig. 2

In the case of the anomalous skin effect, the in-

[2] In the limiting cases (5), Eqs. (3) and (6) can be solved exactly by the Hartmann-Luttinger method.

[3] We take the values of the roots corresponding to the positive real part of the impedance.

crement due to diffuseness begins to decreases when $|l/\delta| \gg (p_0 d)^{1/2} \ln p_0 d$, owing to the smallness of the glancing angle $p_x/p_0 \sim |\delta/l| \ll 1$ of the electrons taking part in the current. Such electrons "see" the surface at a small angle, as a result of which the influence of the roughness is diminished. This is precisely the reason why the result (8.1, 8.2) contradicts the known fact that there is a numerical difference (8/9 and 1) between the values of the impedance at $\rho = 0$ and $\rho = 1$.

In the region $|l/\delta| \sim (p_0 d)^{1/2} \ln p_0 d$, the increment ΔZ is maximal, and the conditions $p_0 d \gg 1$ and $p_x a \sim |\delta/l| a p_0 \gg (p_0 d)^{1/4}$ no longer suffice to be able to iterate with respect to $\sigma(kk')$. It is necessary to satisfy the stronger inequality $\Delta Z/Z_0 \sim (ap_0)^2/p_0 d \ln p_0 d \ll 1$ (this being due to the presence of the sharp peak of $f(p)$ at $p_x = 0$ (see (4)); this inequality can be obtained by taking into account the fact that the roughness has little effect only when $a/d \ll p_x/p_0 \sim |\delta/l|$. In the opposite case we get $\Delta Z \sim Z_0$.

The difficulties involved in experimentally verifying Fig. 2 lie in the need for separating ΔZ, but if the $Z_0(\omega)$ is known this should not be a serious obstacle. It is thus possible to determine the electronic characteristics (even the isotropic ones - Pippard's idea) from Z_0, and the parameters a, d, and ϵ_0 from ΔZ.

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GRAVITATIONAL FIELDS OF THE THIRD TYPE

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In accordance with the classification proposed by me earlier ([1], Sec. 18), a special role is played in Einstein's gravitation equations by gravitational field of the third type. Whereas gravitational fields of the first type include practicall all the physically interpretable solutions of the field equations (the Schwarzschild solution, static field, etc.), and among the known solutions of the second type are fields with cylindrical waves [2], gravitational fields of the third type do not lend themselves as yet to a physical interpretation. Moreover, even a determination of the formal solutions belonging to this class of field is a problem of considerable technical difficulty. However, if we start from the premise that Einstein's theory of gravitation is logically closed and requires no additional definition capable of excluding fields of the third type, such fields are worthy of special attention.

The existence of such fields was predicted before concrete examples were presented. The first example of such a field was apparently the metric obtained by me in 1955 (unpublished

$$ds^2 = e^{-x^2}[e^{-2x^4}(dx^1)^2 + (dx^2)^2] - 2dx^3 dx^4 + x^2(x^3 + e^{x^2})(dx^4)^2$$