crement due to diffuseness begins to decreases when $|l/\delta| \gg (p_0 d)^{1/2} \ln p_0 d$, owing to the smallness of the glancing angle $p_x/p_0 \sim |\delta/l| \ll 1$ of the electrons taking part in the current. Such electrons "see" the surface at a small angle, as a result of which the influence of the roughness is diminished. This is precisely the reason why the result (8.1, 8.2) contradicts the known fact that there is a numerical difference (8/9 and 1) between the values of the impedance at $\rho = 0$ and $\rho = 1$.

In the region $|l/\delta| \sim (p_0 d)^{1/2} \ln p_0 d$, the increment ΔZ is maximal, and the conditions $p_0 d \gg 1$ and $p_x a \sim |\delta/l| a p_0 \gg (p_0 d)^{1/4}$ no longer suffice to be able to iterate with respect to $\sigma(kk')$. It is necessary to satisfy the stronger inequality $\Delta Z/Z_0 \sim (ap_0)^2/p_0 d \ln p_0 d \ll 1$ (this being due to the presence of the sharp peak of $f(p)$ at $p_x = 0$ (see (4)); this inequality can be obtained by taking into account the fact that the roughness has little effect only when $a/d \ll p_x/p_0 \sim |\delta/l|$. In the opposite case we get $\Delta Z \sim Z_0$.

The difficulties involved in experimentally verifying Fig. 2 lie in the need for separating ΔZ, but if the $Z_0(\omega)$ is known this should not be a serious obstacle. It is thus possible to determine the electronic characteristics (even the isotropic ones - Pippard's idea) from Z_0, and the parameters a, d, and ϵ_0 from ΔZ.

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[1] G. E. Reuter and E. H. Sondheimer, Proc. Roy. Soc. A195, 336 (1949).
[2] J. M. Ziman, Electrons and Phonons, Oxford, 1960.
[3] R. F. Greene, Phys. Rev. 141, 687 (1966).
[4] L. A. Fal'kovskii, Zh. Eksp. Teor. Fiz. 53, No. 5 (1970) [Sov. Phys.-JETP 31, No. 5 (1970)].
[5] L. E. Hartmann and J. M. Luttinger, Phys. Rev. 151, 430 (1966).

GRAVITATIONAL FIELDS OF THE THIRD TYPE

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In accordance with the classification proposed by me earlier ([1], Sec. 18), a special role is played in Einstein's gravitation equations by gravitational field of the third type. Whereas gravitational fields of the first type include practicall all the physically interpretable solutions of the field equations (the Schwarzschild solution, static field, etc.), and among the known solutions of the second type are fields with cylindrical waves [2], gravitational fields of the third type do not lend themselves as yet to a physical interpretation. Moreover, even a determination of the formal solutions belonging to this class of field is a problem of considerable technical difficulty. However, if we start from the premise that Einstein's theory of gravitation is logically closed and requires no additional definition capable of excluding fields of the third type, such fields are worthy of special attention.

The existence of such fields was predicted before concrete examples were presented. The first example of such a field was apparently the metric obtained by me in 1955 (unpublished

$$ds^2 = e^{-x^2}[e^{-2x^4}(dx^1)^2 + (dx^2)^2] - 2dx^3 dx^4 + x^2(x^3 + e^{x^2})(dx^4)^2$$

which satisfies the field equations $R_{\alpha\beta} = 0$ and admits a non-Abelian two-term group of motions. It was later observed ([1], Sec. 30) that there exists a two-parameter family of gravitational fields of the third kind having the same group of motions, in vacuum ($R_{\alpha\beta} = 0$)

$$ds^2 = e[\alpha(dx^1)^2 + (dx^2)^2] + 2dx^3dx^4 + \lambda(dx^4)^2,$$

$$\alpha = e(x^2)^2, \quad \lambda = 2\left[x^3 + \frac{e}{4}(x^2)\right]^2 \ln(px^2) - e(x^2)^2 + q,$$

$$p, q = \text{const}, \quad e = \pm 1,$$

and the statement was made that this is a space of maximum mobility of the third type in vacuum. However, an arithmetic error has crept into the derivation of this statement, which entails laborious calculations, so that in fact these gravitational fields admit of a three-member group of motions. This was pointed out by Kollinson and French [3], who, without indicating the metrics, made a remark concerning the possibility of a three-member group. The fields of greatest physical interest among the gravitational fields are as a rule those admitting various symmetries, and particularly groups of motions. Therefore a determination of fields with maximum mobility is certainly worthy of special attention. An example of such a space was found by V. R. Kaigorodov. In a special coordinate system, the metric takes the form

$$ds^2 = -x^3(dx^1)^2 + 2dx^1dx^2 - \frac{(3x^2)^2}{(2x^3)^3}[(dx^3)^2 + (dx^4)^2];$$

it satisfies the field equations $R_{\alpha\beta} = 0$ and admits of a three-member group of motions with a group structure

$$[X_1, X_2] = 0, \quad [X_2, X_3] = -2X_2, \quad [X_3, X_1] = -X_1$$

and group motion operators

$$X_1 = p_1, \quad X_2 = p_4, \quad X_3 = x^1p_1 - x^2p_2 - 2x^3p_3 - 2x^4p_4.$$

An investigation of the λ matrix ([1], Sec. 18) $(R_{AB} - \lambda g_{AB})$, where $R_{AB} \to R_{\alpha\beta\gamma\delta}$, $g_{AB} \to g_{\alpha\gamma}g_{\beta\delta} - g_{\alpha\delta}g_{\beta\gamma}$; $A, B, \ldots = 1, \ldots, 6 \equiv \{14, 24, 34, 23, 31, 12\}$, shows that it has the characteristic $[(3^\circ, 3)]$, i.e., this metric actually defines a gravitational field of the third kind.

The question whether other fields of the third type with group G_3 are admissible is presently under study.

It is remarkable that the field equations $R_{\alpha\beta} = \kappa g_{\alpha\beta}$ of the third type, with $\kappa > 0$, admit of a group of motions G_4 ([1], Sec. 30), and when $\kappa \to 0$ the mobility is lowered, although it is still difficult to interpret this fact physically.

[1] A. Z. Petrov, Novye metody v obshchei teorii otnositel'nosti (New Methods in General Relativity Theory), Nauka, 1966.

[2] A. Einstein and N. Rosen, On Gravitational Waves, J. Franklin Inst., 223 (1937).

[3] C. D. Kollinson and D. C. French, Null Tetrad Approach to Motions in Emply Space-time, J. Math. Phys. 8, No. 4 (1967).