and (3) difficult.

The structure of the spectrum should depend significantly on the impurity concentration or the free-carrier concentration, if it is due to the spin vibrations of the "strings." The line widths are determined mainly by the scatter of the lengths (see formula (1)) or by the intrinsic damping of the spin waves $\gamma \sim Jk_n^4$ (cf., e.g., [3]) if the scatter of the lengths is small.

Finally, when the temperature rises the number of fluctuation spin flips increases, and this leads to a line broadening on the order of $J_1 k_n \exp(-J_1/T)$ (under the condition that $k_n \exp(J_1/T) \gg 1$).

Assuming that the EPR line structure observed in [2] is connected with standing spin waves, we obtain for I a value $\sim$100°K and $k_n \sim 10^{-3}$. This means that approximately 10^{-3} of the total number of free valence bonds are filled, in agreement with the independently measured impurity and dislocation concentrations. The experiment of [2] was performed at very low impurity concentrations. By increasing the number of impurities one can arrive at Read's case, where the bond filling coefficient is approximately equal to 0.1. In this case the spectrum lies in the infrared region (with the exception, of course, of the line $k_n = 0$).

We note that the charges on the dislocation line break the three-dimensional grid of coupled spins and prevent occurrence of "dislocation" ferromagnetism (Sharp and Avery [4], Kosevich and Shklovskii [5]).

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CONCERNING A BOUND NEUTRON IN MATTER

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1. This paper is devoted to an analysis demonstrating the possibility of existence of a long-lived bound state of a neutron in matter. Such a state arises under certain conditions in an irregular medium at low temperature as a result of ordinary nuclear interaction when the collective effects of the medium are taken into account.

The direct impetus for the analysis was the reported [1] experimental observation of neutrons emitted with a delay on the order of several times ten seconds (after the cessation of the neutron exposure) from a previously irradiated LiF crystal at helium temperature. The authors start from the premise that this effect is due to the appearance of a bound state of the neutron with the electron of an F-center due to a Foldy-type interaction. However, even a cursory analysis shows that this interaction is very weak, and is at any rate small compared with the magnetic interaction of the neutron with the spin and orbital electron current. But

even the magnetic interaction is so weak, that in itself it cannot lead to a bound state of a neutron with an electron. The experimental results themselves have raised certain doubts. The analysis below does not lift these doubts, and a quantitative interpretation result turns out to be impossible.

2. Lifetime of the "bound" neutron in a crystal. No matter what mechanism produces the bound state when account is taken of the collective effects of the medium (see the next section), we always have as the effective potential

$$V = \frac{2\pi\hbar^2}{m} \frac{1}{\Omega_0} (-f), \tag{1}$$

where f is the amplitude of the coherent scattering and Ω_0 the volume of the unit cell.

In ordinary cases $|V| \sim 10^{-7}$ eV. The binding energy E_0 can obviously be only of the same order. This raises the question of the lifetime of the bound state in a crystal whose temperature is high compared with E_0.

In diamagnetic matter, the transition of the neutron from the bound state to the continuous spectrum, at any value of E_0, can occur only if phonons are absorbed. We represent the wave function of the neutron in the bound state in the form

$$\psi_0(r) = \sum_k C_k e^{ikr} \Omega^{-1/2} \qquad \sum_k |C_k|^2 = 1 , \tag{2}$$

We then have for the probability of the transition to the continuous spectrum with absorption of one phonon

$$W_0 \cong \frac{2\pi}{\hbar} \left| \frac{2\pi\hbar^2}{m} \right|^2 \Omega_0 \sum_\alpha \int \frac{d^3q}{(2\pi)^3} \int \frac{d^3p}{(2\pi)^3} \frac{1}{\Omega} \left| \sum_k C_k \sum_{m,j} e^{i(k-p+q)R_{mj}} \times \right.$$
$$\left. \times \frac{f_{jm}}{\sqrt{M_j}} (e_{jq\alpha} p - k) \right|^2 \frac{\hbar n_{q\alpha}}{2\omega_{q\alpha}} \delta\left(\hbar\omega_{q\alpha} - \frac{\hbar^2 p^2}{2m} - |E_0|\right) . \tag{3}$$

Here m and j are the indices of the unit cell and of the atom in it, $\vec{q}$ and α are the wave vector of the phonon and the number of the branch, and $n_{q\alpha}$ is the phonon equilibrium distribution function.

At low temperatures, the energy conservation law in (3) leads to the condition $p \gg q$. On the other hand, owing to the great diffuseness of the function ϕ_0, we have for the typical momenta $p \gg k$. The main contribution to the single-phonon transition is then made by incoherent inelastic scattering:

$$W_0 \cong 4\sqrt{2}\pi(\hbar m)^{1/2} \sum_j \frac{|f_j'|^2}{M_j} \sum_m |\psi_0(R_m)|^2 \int d\omega g(\omega) \omega^{1/2} n(\omega) . \tag{3'}$$

Here f_j' is the incoherent part of the scattering amplitude. The coherent inelastic scattering makes a contribution to (3) only by virtue of the spatial inhomogeneity of ψ_0. A direct analysis then leads to an expression of the type (3') with

$$|f_j'|^2 \sim |f_j|^2 4\pi/(p_T \ell)(p_T^3 \Omega_0) ,$$

where ℓ is the characteristic dimension of the localization of $\psi_0(\vec{r})$ and $p_T = \sqrt{2mT}/\hbar$. At temperatures on the order of thos of helium and above, the multiplier of $|f_j|^2$ is noticeably smaller than 1. Assuming for the frequency density function of the phonon spectrum $g(\omega)$ the usual low-frequency representation $g(\omega) = 3\omega^2/\omega_D^3$, with $h\omega_D = \theta_D$, we get finally

$$W_{0\,\mathrm{incoh}} \cong 2\cdot 10^2 (m\theta_D)^{1/2} \sum_j \frac{|f_j'|^2}{M_j} \left(\sum_m |\psi_0(Rm)|^2\right) \left(\frac{T}{\theta_D}\right)^{7/2}. \qquad (4)$$

$$W_{0\,\mathrm{coh}} \sim 3\cdot 10^2 (m\theta_D)^{1/2} \sum_j \frac{|f_j|^2}{M_j} \left(\frac{\hbar^2}{m\theta_D}\right)^2 \frac{1}{\Omega_0^2 \ell} \left(\frac{T}{\theta_D}\right)^{3/2}.$$

The estimate (4) shows that the lifetime of a neutron in the bound state at helium temperatures can amount to tens and hundreds of seconds. This result does not depend on E_0 and consequently it can be stated that the neutron will stay in the bound state for such a long time at an arbitrarily low binding energy.

3. Bound state of a neutron in a crystal. If the scattering length $b = -f$ is positive, as is the case for most nuclei, then there is no bound state in a regular crystal. It can arise, however, in an irregular crystal in the presence of large multivacancies, pores, crystallite boundaries, etc.

Let us consider a pore of spherical form. When $V > 0$, it constitutes a potential well for the neutron. The critical dimension of such a pore, corresponding to the appearance of the bound state, is determined by the known relation

$$R_k = \frac{\pi\hbar}{\sqrt{8mV}}. \qquad (5)$$

$R_c \approx 2 \times 10^{-6}$ cm if $V \sim 10^{-7}$ eV.

Thus, a pore with dimension $R \geq R_c$ is actually a trap for a neutron with low binding energy $E_0 < V$. A similar trap is produced for the neutron following precipitation of a phase in which V_1 (Eq. (1)) is smaller than in the matrix, V_0. Then $R_c = \pi\hbar[8m(V_0 - V_1)]^{-1/2}$. Obviously, the most convenient case is the one in which the precipitated phase has $b < 0$. Special interest attaches to extended defects, such as crystallite boundaries or microcracks, since bound states can exist in them even when the transverse dimension is small compared with R_c.

If the scattering length is negative, then the entire crystal is already a trap for the neutrons. We have in mind a shallow and broad well (of depth $V_0 = -2\pi\hbar^2|b|/m\Omega_0$). Since its linear dimension is large compared with (5), we can use for the number of levels in such a well the quasiclassical expression

$$N_1 = \Omega \frac{1}{6\pi^2} \left(\frac{2m|V_0|}{\hbar^2}\right)^{3/2}, \qquad (6)$$

where Ω is the volume of the crystal. For $|V_0| \approx 10^{-7}$ eV we have $N_1 \sim 10^{16}$. We note that the situation in such a well has much in common with total internal reflection when the neutrons are incident on the crystal from the outside [2].

4. Capture of neutrons in trap. We assume that a stream of monochromatic neutrons of

energy $\varepsilon_0 = h^2 p_0^2/2m < \theta_D$ and let the crystal thickness be limited by the condition that there be no double inelastic collisions. Then the entry of the neutron into the trap will be connected with emission of a phonon having an energy close to ε_0. The probability W_1 of such a process, corresponding to a density of one neutron per cm^3, is given by expression (3) with the substitution $n_{q\alpha} \to (1 + n_{q\alpha})$ and with the integral $(2\pi)^{-3}\int d^3p$ removed. If a neutron flux I_0 is incident on a crystal with N_1 possible bound states, then the number of neutrons ν that falls in the trap per unit time is given by

$$\nu = W_1 I_0 \frac{m}{\hbar p_0} \cong (2\pi)^3 I_0 N_1 \frac{\hbar}{p_0} \sum_l \frac{|f_l'|^2}{M_l} \left(\sum_m |\psi_0(R_m)|^2 g(\omega = \frac{\varepsilon_0}{\hbar}) \right). \tag{7}$$

The energy is best dissipated by incoherent inelastic scattering. Such a scattering will occur for noticeable values of p_0 also for coherently scattering nuclei if the medium is irregular. In this case f_j' in (7) must be taken to mean the coherent amplitude f_j. For a flux $I_0 \sim 10^{10}$ we get from (7) $\nu \sim (10^{-15} - 10^{-16})N_1$ sec^{-1}.

5. Neutron absorption. Let us determine the probability W_a of the n-γ reaction for a neutron in the bound state. For each partial wave in (2) we have the usual transition matrix element, which does not depend on the value of k at small $|\vec{k}|$. Then

$$W_a = \sigma_a(k) \frac{\hbar k}{m} \sum_m |\psi_0(\bar{R}_m)|^2 , \tag{8}$$

where σ_a is the cross section of the n-γ reaction per unit cell ($\sigma_a(k)k = \text{const}$).

The coherent-scattering cross section does not enter in the absorption. But in the case of a bound state in a nonmagnetic medium, the spin interaction of the neutron with the nuclei will likewise not lead to "absorption," since the incoherent process of the spin flip of the neutron and of one of the nuclei simultaneously leaves the neutron in a bound state, since it is impossible to transfer to the neutron an energy on the order of E_0.

The absorption of a neutron from the bound state will thus be determined only by that part of the total cross section, which corresponds to the n-γ reaction. The strongest limitation on the lifetime in the bound state will apparently be imposed just by the value of W_0.

6. Experimental aspects. Expressions (7) and (8) contain as a factor the sum of the squares of the wave function at the sites of the nuclei of the medium. Owing to the delocalization of the wave function in the case of binding in a narrow well, this factor naturally differs from zero also in the case when the trap is a small pore. The requirements on the magnitude of this factor are then contradictory from the point of view of increasing the capture by the bound state and the lifetime of this state. In this connection, when $b > 0$ the optimal medium is porous with pore dimensions on the order of R_c [Eq. (5)]. Such a medium constitutes a unique "sponge" that absorbs continuously neutrons with a characteristic time equal to the lifetime of the bound state. From the point of view of energy dissipation, the most convenient is a medium in the form of an amorphous powder. Under optimally selected conditions and at $I_0 \sim 10^{10}$, approximately one or several neutrons per cm^3 can fall in such

a sponge each second. By varying the dimensions of the pores and by the same token the factor $\sum_m |\psi_0(R_m)|^2$, it is possible to vary the ratio of the capture time to the lifetime of the bound state.

When $b < 0$, a regular medium is optimal. In this case, using the value of N_1 [Eq. (6)], we obtain approximately the same estimate for the number of neturons captured in one cm^3 of the crystal. By varying the temperature, we can easily obtain an arbitrary ratio of $W_0(T)$ (4) to W_a (8). This yields direct experimental possibilities of investigating long-lived neutrons in a crystal.

As to the experiments with LiF [1], it is impossible to explain the experimental data quantitatively, primarily because of the large absorption cross section of Li, although qualitatively the appearance of the bound state might be attributed in this case to the presence of large-scale defects such as pores or a precipitated phase of Li. The value of $\tau_a = 1/W_a$ can be raised to a time on the order of one second only by assuming that the crystal contains large cavities (or cracks) with dimensions larger by several orders of magnitude than given by (5). But then, in the case of a vacuum cavity, the time of capture in the bound state increases sharply. The picture is therefore internally contradictory.

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