magnetoresistance increase sharply (Fig. 2a). The magnetic-field region where the condition $\mu H \ll 1$ is satisfied narrows down at $p = p^*$ from 20 - 30 Oe ($p = 1$ bar) to 1 - 2 Oe, from which it follows that at $p = p^*$ the carrier mobilities in the investigated alloys greatly exceed the mobility in pure bismuth [9]. Figure 2b shows the pressure dependence of the electron mobility in L, for an alloy with $x = 0.15$ (n-type), as calculated from the components of the galvanomagnetic tensor.

6. The approach and "reflection" of the extrema at the point L in the $Bi_{1-x}Sb_x$ alloys was observed also in strong magnetic fields (semiconductor - "quasimetal" - semiconductor transitions in a magnetic field) [3]. The longitudinal magnetoresistance passed in this case through a maximum (in a field $H = H_c$), dropped to a value much lower than the value at $H = 0$, and then increased exponentially. The fields required to observe transitions of this type at $\epsilon_g \approx 6$ meV were of the order of 200 - 300 kOe. By applying pressure, it was possible to obtain semiconducting $Bi_{1-x}Sb_x$ with very small ϵ_g and to effect transitions of the indicated type in constant fields of several dozen kOe. Figure 3 shows by way of an example a plot of the longitudinal magnetoresistance against H for $Bi_{1-x}Sb_x$ alloys with $x = 0.089$ at a pressure $p \sim p^*$ and a plot of H_c against p. We see that with increasing p H_c passes through a minimum, thus indicating directly the approach and "reflection" of the bands in L under the influence of the pressure. The character of the obtained relations is in splendid correlation with the results obtained in pulsed fields [3], and is convincing proof of the correctness of their interpretation.

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ELECTRON ENERGY LEVELS AT $Z > 137$

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Submitted 4 January 1970
ZhETF Pis. Red. 11, No. 5, 254 - 256 (5 March 1970)

As is well known, the solution of the Dirac equation in the field of a point charge Ze is mathematically correct only if $Z < 137$. The energy of the lower level of the discrete spectrum ($n = 1$, $j = 1/2$) is

$$\epsilon_0 = \sqrt{1 - \alpha^2} \qquad (\alpha = Ze^2 = Z/137, \qquad \hbar = c = m = 1). \tag{1}$$

ϵ_0 reaches zero when $\alpha = 1$, and the continuation of formula (1) to the region $\alpha > 1$ leads to imaginary values of ϵ_0. When $\alpha > 1$ it is therefore necessary to take into account the finite dimensions of the nucleus. Such a formulation of the problem belongs to Pomeranchuk and Smorodinskii [1], who gave a correct description (from the qualitative point of view) of

the phenomena occurring when $Z > 137$, namely, the levels continue to drop with increasing α until the level $1S_{1/2}$ reaches the boundary $\epsilon = -1$ of the lower continuum at a certain "critical" value $\alpha_{cr} > 1$. The derivation of the equation for α_{cr} in [1], however, contains an error [1)], so that the values of $Z_{cr} = 137\alpha_{cr}$ given there are overestimated. The charge Z_{cr} determines the "electrodynamic boundary" of the periodic system of elements. Recently, in connection with progress in the synthesis of superheavy nuclei, the question of the maximum possible Z has become more urgent [2].

To find α_{cr}, it suffices to solve the Dirac equation at $\epsilon = -1$ in the potential

$$V(r) = \begin{cases} -\alpha/r, & \text{if} \quad r > R \\ -\frac{\alpha}{R} f\left(\frac{r}{R}\right), & \text{if} \quad 0 < r < R \end{cases} . \tag{2}$$

The form of the cutoff function $f(x)$ depends on the distribution of the electric charge over the volume of the nucleus. Thus, $f(x) = (1/2)(3 - x^2)$ corresponds to a constant density, and $f(x) \equiv 1$ corresponds to the concentration of the entire charge on the surface of the nucleus. It can be shown that when $r > R$ the solution takes the form

$$r g(r) = K_{i\nu}(\sqrt{8\alpha r}), \qquad f(r) = \alpha^{-1} r g'(r) \tag{3}$$

(for the level $1S_{1/2}$). Here $\nu = 2\sqrt{\alpha^2 - 1}$, $K_{i\nu}$ is the MacDonald function with imaginary index (for which tables are given in [3]), and g and f are radial functions for the upper and lower components of the Dirac bispinor. In the internal region $r < R$, we use the smallness of R compared with the Compton wavelength of the electron. It is convenient to change over to the function $\xi = rg/(rg)'$, which satisfies the equation ($x = r/R$):

$$\frac{d\xi}{dx} = 1 + \alpha^2 f^2 \xi^2 + \frac{f'}{xf}\xi(\xi - x); \qquad \xi(0) = 0 . \tag{4}$$

The joining of the two solutions at $r = R$ yields an equation for the determination of α_{cr}:

$$zK'_{i\nu}(z) - 2\lambda K_{i\nu}(z) = 0, \tag{5}$$

where $z = \sqrt{8\alpha R}$ and $\lambda = [\xi(1)]^{-1}$. To find $\lambda = \lambda(\alpha)$ it is necessary first to solve Eq. (4), which can be done by numerical methods. In the simplest case $f(x) \equiv 1$ this solution is obtained analytically: $\xi(x) = (\tan \alpha x)/\alpha$, $\lambda = \alpha \cot \alpha$. The values of α_{cr} obtained from (5) are listed in the table for two cutoff models: the charge is on the surface of the nucleus (column I) and is uniformly distributed over the volume of the nucleus (column II). In column III are given for comparison the values of α_{cr} indicated in [1] (we note that only model I with $f(x) \equiv 1$ was considered there).

[1)] No account is taken of the absence of nodes in the wave function of the ground state.

$R \cdot 10^{12}$ cm	α_{cr}		
	I	II	III
0.8	1.248	1.224	1.28
1.0	1.271	1.243	–
1.2	1.291	1.260	1.46

If we extrapolate the function $R = r_0 A^{1/3}$ to the region $Z > 137$, assuming (as for heavy nuclei) that $r_0 = 1.1 \times 10^{-13}$ cm and $A = 2.5Z$, then the critical charge of the nucleus is found to be $Z_{cr} = 168$ (in model II; in this case $R_{cr} = 8.25$ F). This value of Z_{cr} is not very sensitive to details of the distribution of the charge over the nucleus. Thus, the use of model I yields $Z_{cr} = 171$.

This raises the question of what happens when $Z > Z_{cr}$. One can visualize a hypothetical experiment in which the coalescence of two bare subcritical nuclei with charge $Z/2 < Z_{cr}$ each gives rise to a superheavy nucleus with a charge Z larger than critical (as indicated in [4], this is essentially the situation that arises also when such nuclei approach each other to within a distance $\lesssim 1$). In accordance with [4], spontaneous production of pairs by the Coulomb field begins when $\alpha > \alpha_{cr}$, but the details of this process are not as proposed in [4]. First, we call attention to the fact that the electron state at the edge of the lower continuum is localized (unlike the case $\epsilon = +1$). Indeed, from (3) we have g, $f \sim \exp(-\sqrt{8\alpha r})$ when $r \to \infty$. This property is typical of the relativistic Coulomb problem and does not depend on the spin. To understand its cause, let us consider the Klein-Gordon equation, which is mathematically equivalent to the nonrelativistic Schrodinger equation with an effective energy $E = (\epsilon^2 - 1)/2$ and potential $U = \epsilon V - V^2/2$. When $V = -\alpha/r$, the "tail" of the potential U is of the form $U \sim -\epsilon\alpha/r$, i.e., its sign depends on the sign of the energy ϵ. From this we get the asymptotic form of the wave function as $r \to \infty$:

$$\chi(r) \sim e^{-\chi r} r^n \qquad (\chi = \sqrt{1-\epsilon^2}, \quad n = \epsilon\alpha/\chi) . \tag{6}$$

When $\epsilon \to +1$ and $n \to +\infty$, delocalization of the electron takes place, and when $\epsilon \to -1$, the pre-exponential factor decreases more rapidly than any power of r, in agreement with (3). When α increases above α_{cr}, the bound state (3) turns into a quasistationary state having an asymptotic form of the type of a divergent wave at infinity. Its energy becomes complex, with the imaginary part determining the probability w of pair production per unit time. We present an expression for w in the limiting case of very small R:

$$w = \frac{6\pi}{5} \exp\left\{ -\sqrt{\frac{5\pi}{3} \frac{\nu_{cr}^2}{\nu - \nu_{cr}}} \right\} \quad (\nu = 2\sqrt{a^2 - 1}). \tag{7}$$

This formula is valid only if $\nu_{cr} \ll 1$, which is not satisfied for real values of R. However, even the more accurate formula for w, obtained without assuming ν to be small, is

quite similar qualitatively to (7). Near $\alpha = \alpha_{cr}$ (the pair production threshold), the probability of w is exponentially small, $w \sim \exp[-\sqrt{a/(\alpha - \alpha_{cr})}]$. We emphasize that the exponential vanishing of w at the threshold is a purely Coulomb effect: a static field produces pairs only in the region where $|V(r)| > 2$, and then the positrons go through the Coulomb barrier $U(r) = \alpha/r - (\alpha^2/2r^2)$, the penetrability of which is exponentially small when $\epsilon \to -1$. This smallness is missing in pair production by a field with a finite effective radius. Thus, for example, for a square well with radius r_0 and depth V we get $V_{cr} = (1 + \pi^2 r_0^{-2})^{1/2}$ and $w \sim (V - V_{cr})^{3/2}$ as $V \to V_{cr}$.

The author is deeply grateful to A. M. Perelomov and M. V. Terent'ev for numerous discussions during the work, and also to S. S. Gershtein, Ya. B. Zel'dovich, B. L. Ioffe, L. B. Okun', and V. V. Sudakov for a discussion of the results and a number of valuable remarks.

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MECHANISM OF FORMATION OF NEGATIVE RESISTANCE IN SEMICONDUCTORS DURING IMPURITY BREAKDOWN

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Submitted 12 January 1970
ZhETF Pis. Red. 11, No. 5, 257 - 260 (5 March 1970)

In certain semiconductors, an S-shaped current-voltage characteristic (with two values of the voltage) is produced upon impact ionization of the impurity [1,2]. The existing theories of impact ionization result in a unique dependence of the current on the voltage. Negative differential resistance (NDR) arises if the dependence of the temperature T_e of the hot electrons (the existence of which is henceforth assumed) or of their concentration n on the electric field E becomes doubly-valued. In this communication we consider certain mechanisms that lead to such dependences.

1. NDR due to lack of phonon equilibrium. To determine the current-voltage characteristic in a strong electric field at low temperatures it is necessary to take into account the lack of phonon equilibrium [3,4]. This phenomenon turns out to be particularly important in impact ionization. Under breakdown conditions, the concentration of the conduction electrons, and consequently the Joule power, greatly increases, and the rate of transfer of this poer to the lattice, as a result of the non-equilibrium of the phonons, does not depend on the electron concentration but remains constant. As a result, a superheat instability sets in, i.e., a doubly-valued dependence of the electron temperature on the electric field.

The $T_e(E)$ dependence is determined by the energy balance equation

$$\sigma E^2 = W_f + W_{ir}, \tag{1}$$

where W_f describes the loss of energy to the lattice as the result of electron-phonon interaction, and W_{ir} is due to impact ionization and recombination. The hot electrons give up