quite similar qualitatively to (7). Near $\alpha = \alpha_{cr}$ (the pair production threshold), the probability of w is exponentially small, $w \sim \exp[-\sqrt{a/(\alpha - \alpha_{cr})}]$. We emphasize that the exponential vanishing of w at the threshold is a purely Coulomb effect: a static field produces pairs only in the region where $|V(r)| > 2$, and then the positrons go through the Coulomb barrier $U(r) = \alpha/r - (\alpha^2/2r^2)$, the penetrability of which is exponentially small when $\epsilon \to -1$. This smallness is missing in pair production by a field with a finite effective radius. Thus, for example, for a square well with radius r_0 and depth V we get $V_{cr} = (1 + \pi^2 r_0^{-2})^{1/2}$ and $w \sim (V - V_{cr})^{3/2}$ as $V \to V_{cr}$.

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MECHANISM OF FORMATION OF NEGATIVE RESISTANCE IN SEMICONDUCTORS DURING IMPURITY BREAKDOWN

A. N. Zaitsev, A. K. Zvezdin, and V. V. Osipov
Moscow Institute of Radio Engineering, Electronics, and Automation
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In certain semiconductors, an S-shaped current-voltage characteristic (with two values of the voltage) is produced upon impact ionization of the impurity [1,2]. The existing theories of impact ionization result in a unique dependence of the current on the voltage. Negative differential resistance (NDR) arises if the dependence of the temperature T_e of the hot electrons (the existence of which is henceforth assumed) or of their concentration n on the electric field E becomes doubly-valued. In this communication we consider certain mechanisms that lead to such dependences.

1. NDR due to lack of phonon equilibrium. To determine the current-voltage characteristic in a strong electric field at low temperatures it is necessary to take into account the lack of phonon equilibrium [3,4]. This phenomenon turns out to be particularly important in impact ionization. Under breakdown conditions, the concentration of the conduction electrons, and consequently the Joule power, greatly increases, and the rate of transfer of this poer to the lattice, as a result of the non-equilibrium of the phonons, does not depend on the electron concentration but remains constant. As a result, a superheat instability sets in, i.e., a doubly-valued dependence of the electron temperature on the electric field.

The $T_e(E)$ dependence is determined by the energy balance equation

$$\sigma E^2 = W_f + W_{ir}, \tag{1}$$

where W_f describes the loss of energy to the lattice as the result of electron-phonon interaction, and W_{ir} is due to impact ionization and recombination. The hot electrons give up energy to the long-wave phonons (LP) with wave number $g \leq (2mT_e/\hbar^2)^{1/2}$. If $T_eT_0 < T_0/8ms^2$, where s is the speed of sound, then the LP relax in turn on the thermal phonons, with a time τ_{ff}. When $\tau_{ff} \gg \tau_{fe}$[1] (τ_{fe} is the time of relaxation of the LP on the electrons), the LP temperature is equal to T_e, and W_f in (1) is the power given up by the LP to the thermal phonons. From the kinetic equation it follows that

$$n = N_d \left[1 + \frac{\sigma_r}{\sigma_i} \exp \frac{I}{T_e}\right]^{-1}, \tag{2}$$

$$W_f = \frac{T_e - T_0}{\tau_{ff}(T_0)} \frac{4}{\pi} \left(\frac{mT_e}{\hbar^2}\right)^{3/2} \left(\frac{T_e}{T_0}\right)^{1/2} = \frac{T_e - T_0}{\tau_{ff}(T_0)} \left(\frac{T_e}{T_0}\right)^{1/2} \Omega(T_e), \tag{3}$$

$$W_{ir} = [I + a(T_e) T_e] n^2 \sigma_r v , \tag{4}$$

where σ_i and σ_r are the ionization and recombination cross sections, v is the thermal velocity of the electron, I is the impurity ionization energy, T_0 is the lattice temperature, and N_d is the impurity concentration. If $W_f \gg W_{ir}$, which is satisfied when $n \ll \Omega(T_e) \cdot (T_e/I) \cdot (\tau_r/\tau_{ff}) = n_2$,[2] the current-voltage characteristic can be written in parametric form

$$\begin{aligned} E &= E_0 \left(\frac{T_e}{I}\right)^{\frac{(3-t)}{2}} \exp \frac{I}{2T_e} , \\ i &= i_0 \left(\frac{T_e}{I}\right)^{\frac{(3+t)}{2}} \exp \frac{I}{2T_e} , \end{aligned} \tag{5}$$

where t determines the dependence of the mobility μ on T_e ($\mu = \mu(T_0)(T_e/T_0)^t$). The current-voltage characteristic calculated from expressions (5) is shown in Fig. 1 (curve 1). We note that no account was taken of the thermal generation in the derivation of (5). In weak fields, thermal generation is significant and the current-voltage characteristic does not differ strongly from the ohmic one (curves 2 - 5, Fig. 1). From (4) it follows that the NDR terminates at $T_e = I/(3 - t)$. It is interesting to note that in the absence of breakdown, the lack of phonon equilibrium leads to a vanishing of the NDR of the superheat type [3, 4].

2. NDR due to energy relaxation in ionization and recombination processes. If the loss of energy of the hot electrons to the lattice is determined by the second term in (1), then the $T_e(E)$ dependence can also be doubly-valued even without allowance for the lack of phonon equilibrium. This is connected with the fact that the recombination time increases rapidly with increasing T_e, and consequently the rate of transfer of the Joule power to the lattice decreases when almost all the donors are ionized.

The condition $W_{ir} \gg W_f$ is satisfied when the recombination time $\tau_r \ll \tau_{ef}$ (τ_{ef} is the relaxation time of the electron energy), if the LP are in equilibrium, or if $\tau_r \ll \tau_{ff}$ ($n > n_2$) if they are not in equilibrium. In this case the current-voltage characteristic is determined

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[1] In Ge at $T_0 \sim 20°$ K, this relation is satisfied when $n > 5 \times 10^{12}$ cm$^3 = n_1$.

[2] In G at $N_d \sim 10^{16}$ cm^3 and $T_0 \sim 50°$ K we have $n_2 \gtrsim 10^{14} - 10^{15}$ cm^3.

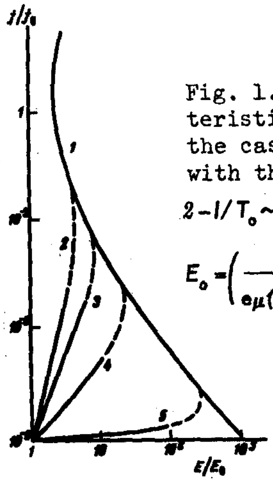

Fig. 1. Current-voltage characteristics of a semiconductor in the case of impurity breakdown, with the phonons in equilibrium: $2 - I/T_0 \sim 5$; $3 - T/T_0 \sim 10$; $5 - T/T_0 \sim 20$;

$$E_0 = \left(\frac{I}{e\mu(I)\,\tau_{ff}(I)}\right)^{1/2}, \quad i_0 = \left(\frac{eI\mu(I)}{\tau_{ff}(I)}\right)^{1/2}$$

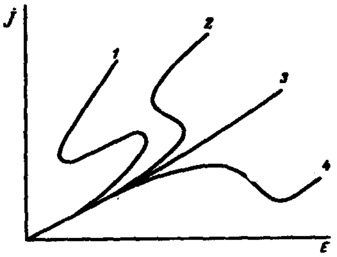

Fig. 2. Current-voltage characteristics of a semiconductor in the case of impurity breakdown, when the relaxation of the electron energy is determined by the ionization and relaxation processes: 1 - $k + t > 0$, $k - t > 0$; 2 - $k + t > 0$, $k - t < 0$; 3 - $k + t < 0$, $k - t < 0$; 4 - $k + t < 0$, $k - t > 0$.

by the expressions:

$$E = \left[\frac{(I + aT_e)}{e\mu(T_e)\,\tau_r(T_e)}\;\frac{n}{N_d}\right]^{1/2}, \quad j = n\left[\frac{(I + a\,T_e)}{\tau_r(T_e)}\,e\mu(T_e)\right]^{1/2}. \tag{6}$$

At sufficiently high temperatures, when n is practically independent of T_e, we have

$$T_e,\ E_k \sim T_e^{-\frac{(t-k)}{2}},\ j \sim T_e^{\frac{(t-k)}{2}} \left(d\,T_e \lesssim I\right),$$

where k determines the dependence of τ_r on T_0 $\left(\tau_r = \tau_i(T_0)\,(T_e/T_0)^k\right)$. Depending on the values of the parameters k and t, four types of current-voltage characteristic are possible, as shown in Fig. 2. For different momentum relaxation mechanisms we have $-1/2 \leq t \leq 3/2$. As to k, different theoretical models and experiment yield for it values in the interval $1/2 \leq k \leq 2.3$. Therefore the characteristic observed in experiment is most likely to be of type 1.

3. NDR due to screening of the impurity potential by the non-equilibrium electrons. A feature of impurity breakdown is the possible occurrence of an S-shaped current-voltage characteristic, owing to the ambiguity in the dependence of the carrier density on the electric field. Such a dependence can result from screening of the impurity potential by non-equilibrium electrons, the concentration of which increases during breakdown, leading to a decrease of the impurity ionization energy [5]. We note that no complete repulsion of the impurity level into the conduction band can occur in our case, for this would require a free-electron density exceeding the critical density corresponding to the Mott transition, which is usually determined by the relation $N_d^{1/3} a \sim 0.25$. It turns out that the possible occurrence of an ambiguous $n(T_e)$ dependence is determined by the value of the parameter $I = (8\pi N_d{}^3 a)^{1/2}$, which, in accordance with the Mott condition, should be smaller than 0.55. Analysis shows that in S-shaped current-voltage characteristic appears only when $\sigma_i > \sigma_r$ and $A \geq 0.2$, i.e., when the densities are close to critical. In a sufficiently strong magnetic field, owing to the deepening of the impurity levels, the impurity concentration corresponding to the Mott transition increases [6]. Therefore the NDR can occur under less stringent conditions on the impurity density (in spite of the smoother dependence of the ionization energy on the screening radius). For example, in InSb at $H \sim 10$ kG, the NDR occurs when $A \gtrsim 5$, whereas $A_{max} \sim 100$.

The NDR appears in relatively weak electric field when $T_eI \sim 0.1 - 0.2$.

Bearing in mind the possibility of experimentally separating these mechanisms, we note that the second mechanism leads to NDR in a strong electric field ($T_e(E) \sim I$), when almost all the impurities are ionized, while the first and third mechanisms appear in a weak field. The third "works" when N_d is close to the density corresponding to the Mott transition.

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PERIPHERAL PARTICLE GENERATION AT HIGH ENERGY

V. A. Kolkunov, E. S. Nikolaevskii, and K. A. Ter-Martirosyan

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It will be shown below that the spectra of π^-, K^-, and $\bar{p}$, obtained [1] by the combined group of CERN and the Institute of High Energy Physics (IHEP) by irradiating Al^{27} nuclei with 70-GeV protons, are well described by the simple model of peripheral production corresponding to the diagram of Fig. 1a. The rapid decrease of the spectra [1] with increasing laboratory energy of the particles is a consequence of the decrease of the vertex G of the transformation of the proton into the system of final particles (π^-, Δ^{++}, or K^- (K^+p) or $\bar{p}(pp)$) - in the region of not very large values of its mass $\sqrt{s_1}$, of the order of the resonant values. Let us consider first the reaction $pZ \to \pi^- \Delta^{++} Z'$, where Z is the Al nucleus (or one its nucleons). The diagram of Fig. 1 corresponds to the amplitude

$$T = \eta_\alpha g_Z(t) G(s_1, u_1; t)\, s^{\alpha(t)}, \tag{1}$$

where $\alpha(t) = 1 + \alpha'(0)t$ is the trajectory of the Pomeranchuk pole, $\eta_\alpha = i - \cot\pi\alpha/2 \approx i \exp(-(i\pi/2)\,\alpha'(0)t)$, the invariants $s_1 t$ and s_1, u_1 are indicated in Fig. 1, and the vertex G is represented in the form

$$G = \tilde{g}(t)\, \frac{e^{-\gamma s_1} + \lambda s_1^{1+\beta(u_1)}}{s_1^{\alpha(t)}(s_1 - m_0^2 + i m_0 \Gamma_0)}, \tag{2}$$

in which it exhibits at small $s_1 \sim m_0^2$ a resonant behavior corresponding to formation [2] and decay1) of a resonance (Fig. 1b) with mass m_0 and width Γ_0 into π^- and Δ^{++}, and $s_1 >> m_0^2$ it has a Regge behavior $G \sim s_1^{\beta(u_1) - \alpha(t)}$, which yields in (1) a multireggeon response $T \sim s_1^{\beta(u_1)} s_2^{\alpha(t)}$ for $s_1 >> m_0^2$ [3] we have $s \approx s_1 s_2 m_\Delta^{-2}$, see Fig. 1c). By γ and λ we denote in (2) certain parameters, and the resonant state is chosen to be M(1410) with I = 1/2, the

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1) No account is taken in the first term of (2) of the dependence of this decay on the direction $\vec{n}_{12}$ in the rest system of the resonance (i.e., on u_1, see Fig. 1). In addition, it is assumed that the dependence on t in G is separated in the form of a factor $\tilde{g}(t) \sim \exp(R_\rho^2 t/2)$.