When $H > H_c$ and in the case of carriers of the same sign,

$$\sigma_1 = \frac{n e c}{H} \; ; \qquad H = \sqrt{H_0^2 + b \ln \frac{f}{r_0}} \; ; \qquad b = 8 \pi n e E_0 r_0^2 \; .$$

The sign of the change of the magnetic field in the presence of a radial current is determined by the sign of the Hall constant, i.e., by the sign of the charge of the majority carriers.

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CORRESPONDENCE OF THE EQUATIONS OF THE MULTIPERIPHERAL AND MULTIREGGEON THEORIES OF INELASTIC PROCESSES

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Submitted 2 February 1970

The multiperipheral (MP) theory of inelastic processes, in which single-pion diagrams are considered (see Fig. 1), is formulated [1, 2] within the framework of the Bethe-Salpeter (BS) equation. In the multireggeon (MR) theory [3], which considers diagrams with exchange of one vacuum reggeon (see Fig. 2), its analog is the equation of Chew, Goldberger, and Low (CGL) [4].

These theories describe inelastic processes that are different from each other, i.e., they are valid in different regions of the phase volume [5, 6]. For most processes, the criteria of applicability of the MR theory are satisfied [5]. Recent papers (see [7, 8]), however, contain phenomenological attempts to extend the applicability of the MR theory to the entire phase volume by introducing "clusters" and exchange of "meson" reggeons (i.e., by replacing Fig. 2a with the diagram of Fig. 2b with $R \neq P$).

We write down a single equation for the inelastic processes, which reduces to the BS equation in most of the phase volume, reduces to an equation of the CGL type in the region of applicability of the MR theory. Such an equation was considered in [9], but only for particular cases - the model of Amati et al. [10] and the completely reggeized model [4] (Fig. 2a), which cannot pretend to describe the majority of the inelastic processes.

We denote by $A_1(p_a, p_b)$ the imaginary part of the amplitude of $0°$ elastic scattering of particles with momenta p_a and p_b in the s-channel. We define [4] the function B as follows:

$$A_1(p_a, p_b) = \frac{1}{4 \pi^4} \int d^4 k_1 \bar{A}_1(p_a, k_1) D^2(k_1^2) B(k_1, p_a, p_b) \; . \tag{1}$$

This relation is valid both in the MP and in the MR, but in the former case $\bar{A}_1$ and D^2 are interpreted as the imaginary part of the irreducible block of the amplitude of the elastic scattering and the square of the pion propagator, and in the MR as the squares of the vertex

Fig. 1. Multiperipheral diagram.

Fig. 2. a) Fully Reggeized diagram, b) multireggeon diagram with production of groups of particles. P - vacuum reggeon.
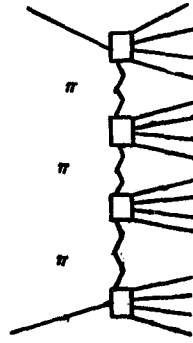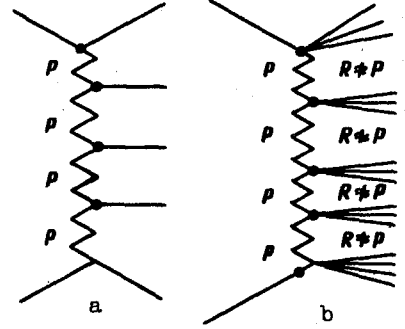

Fig. 1   Fig. 2

part and of the signature factor[1]. In both cases the equation for B can be written in the form

$$B(k_1, p_a, p_b) = \bar{B} + \frac{1}{4\pi^4} \int d^4k_3 \bar{A}_1(k_1, k_3) D^2(k_3^2) R^2(k_3, k_1, p_a) B(k_3, k_1, p_a),\qquad (2)$$

where $\bar{B} = \bar{A}_1(k_1, p_b) R^2(k_1, p_a, p_b)$, R is the Regge factor, and the symbols for the momenta are clear from Fig. 3 (at t = 0 we have $k_1^2 = k_2^2 = k_3^2 = k_4^2$, etc.)

According to usual rules, R must be chosen in the form

$$R_I(k_3, k_1, p_a) = |z_{a3}|^{\alpha(k_1^2)},\qquad (3)$$

where

$$|z_{a3}| \approx \frac{2k_1^2 s_{a3}}{s_{a1} s_{13}} - 1$$

is the cosine of the scattering angle in the t-channel, $s_{ij} = (k_i - k_j)^2$, $s_{ai} = (p_a - k_i)^2$, and $\alpha(k^2)$ is the exchange Regge trajectory.

However, it is sometimes chosen in the form

$$R_{II}(k_3, k_1, p_a) = (s_{a3} / s_0)^{\alpha(k_1^2)},\qquad (4)$$

where s_0 = const. We consider both cases.

In the BS equation, $\alpha \equiv 0$ and $R \equiv 1$. Therefore in the MP B does not depend on p_a. In the MP, both the kernel and the function itself depend only on two vectors, and therefore [1] we can use an expansion in Legendre polynomials. In the MR they depend on three vectors, and this leads to the need [9] of dealing with d-functions that are representations of the group O(2, 1).

Generalizing the results of [9] (see formula (4.19) of [9]) to the case of an arbitrary form of $\bar{A}$, we can easily obtain from (2) an equation for the partial amplitudes b_ℓ^i at any t:

$$b_\ell^i(t, k_1^2, k_2^2) = \bar{b_\ell^i} + \frac{1}{4\pi^3|t|} \sum_n \int dr\, dv [-t(t - 4\mu^2) + 2tr - v^2]^{1/2} \times$$

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[1] In the model [8, 9] of the diagram of Fig. 2a we have $\bar{A}_1 = g^2(p_a^2, k_1^2) \delta^+((p_a - k_1)^2 - m^2)$. The signature factor is so normalized that at pole it goes over into the resonator, apart from the phase.

$$\times \overline{f_{\ell}^{'in}} B_{\ell}^{2}(t, k_3^2, k_4^2) D(k_3^2) D^*(k_4^2) , \tag{5}$$

$$\overline{f_{\ell}^{'in}} = \int\limits_{z_0}^{\infty} dz\, \overline{C}(t, z, k_1^2, k_2^2, r, v) d_{a_i(k_1^2, k_2^2),\ a_n(k_3^2, k_4^2)}^{\ell}(z) , \tag{6}$$

$$r = -k_3^2 - k_4^2 - 2\mu^2 , \quad v = k_4^2 - k_3^2 , \quad \overline{b_{\ell}^{i}} = \overline{f_{\ell}^{'in}}(r = 0, v = 0, a_n = 0). \tag{7}$$

$\alpha_i(k_1^2, k_2^2) = \alpha_i(k_1^2) + \alpha_i(k_2^2)$ (i denotes the concrete Regge trajectory). The explicit form of the d-functions is written out in [9]. At $t = 0$ we have

$$z = (s_{13} + k_1^2 + k_3^2)/2\sqrt{k_1^2 k_3^2} ; \tag{8}$$

z_0 is obtained from z at $s_{13} = 4\mu^2$. The expressions for C and $\overline{C}$ at $t = 0$ take the following form[2]: in variant I see (3)

$$\overline{C}_I = \overline{A}_1(s_{13}, k_1^2, k_3^2) ; \tag{9}$$

in variant II:

$$\overline{C}_{II} = \overline{A}_1(s_{13}, k_1^2, k_3^2)(s_{a1}s_{13}/2k_1^2 s_0)^{2a_i(k_1^2)} \tag{10}$$

Equation (5) makes it possible to replace the difficult problem of the exact equation (2) by the easier problem of finding the asymptotic forms of this solution by investigating the analytic structure of the partial amplitudes in the cross-channel. We are particularly interested in the existence of a solution of (2) with an asymptotically constant cross section. In this case b_{ℓ}^{i} has a pole at the point $\ell(t)$ with $\ell(0) = 1$. The existence condition for the solution of (5) is the absence of such a pole from the kernel $\overline{f_{\ell}^{In}}$. Since the pole of $\overline{f_{\ell}^{In}}$ can also not be located at $\ell > 1$, it must be located at $\ell < 1$.

To determine the position of the leading singularity of $\overline{f_{\ell}^{in}}$ in the ℓ-plane, it is sufficient to consider the contribution made to the integral (6) by large values of z, i.e., s_{13}. If the asymptotic behavior of the irreducible block is given in the form $\overline{A}_1(s_{13}) \sim s_{13}^{v}$ as $s_{13} \to \infty$, where v is a certain constant, then, using the asymptotic form [9] of the d-functions

$$d_{a_i, a_n}^{\ell}(z) \sim (\ell + 1 - 2a_i(k^2))^{-1} z^{-\ell-1} ,$$

we find that the position of the singularity is given by the integral: in the former case (see (3), (9))

$$(\ell + 1 - 2a_i(k^2))^{-1} \int\limits^{\infty} dz\, z^{v-\ell-1} \tag{11}$$

and in the latter case (see (4), (10))

$$(\ell + 1 - 2a_i(k^2))^{-1} \int\limits^{\infty} dz\, z^{v-\ell-1+2a_i} . \tag{12}$$
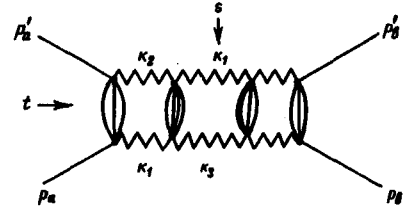
Fig. 3. Shadow elastic scattering

The condition for the existence of a solution of (5) leads to the following requirement in both cases

$$-1 + 2\alpha_i(k^2) < 1 \qquad (13)$$

and in addition, in the first case

$$\nu < 1 \qquad (14)$$

and in the second case

$$\nu + 2\alpha_i(k^2) < 1 \quad . \qquad (15)$$

These conditions explain why difficulties arose with the Pomeranchuk pole in the MP [11] and in the MR [6, 4]. In the BS equation $\alpha \equiv 0$ and the conditions (14) and (15) are violated at $\nu = 1$, i.e., when the asymptotically constant cross section is chosen in the irreducible block. In the CGL equation the conditions (14) and (15) are not violated, owing to the δ-function form of $\overline{A}_1$, but condition (13) is violated at the point $k^2 = 0$ if α_i is a vacuum pole trajectory. The difficulty can be eliminated in three ways: 1) by excluding the point $k^2 = 0$ from consideration, stipulating that the constants of coupling of two vacuum reggeons with the particle vanish at this point [6] (see the weak-coupling model [12]): 2) by requiring that the total cross section decrease ($\alpha_p(0) < 1$) [4]: 3) by assuming a weaker singularity (than a pole) at $\ell = 1$ [11].

We emphasize once more that the first case, which is most realistic, reduces to the BS equation, since the integrand in (11) does not depend on α_i.

Even in the second case, the difference between the MP and the MR drops out from Eq. (5) obtained after integrating over the Treiman-Yang angle, since the dependence on s_{a3} in (4) reduces to a dependence on the product $s_{a1}s_{13}$ in (10), i.e., to a choice of definite form factors in the vertices of the multiperipheral chain.

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