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"MIXMASTER UNIVERSE" AND THE COLD VARIANT

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Recently Misner [1, 2] raised the question of a cosmological model in which the observed isotropy of the universe (in particular, the isotropy of the relict radiation [3] might have a natural explanation. In other words, one seeks in the initial stage at high density a more general anisotropic and inhomogeneous solution that approaches asymptotically in the course of time Friedmann's homogeneous and isotropic solution observed at the present time.

Misner proposed that such properties is possessed by the solution of Belinskii, Lifshitz, and Khalatnikov [4, 5], which they obtained during the course of an investigation of the singularity of the equations of general relativity theory.

The initial solution is homogeneous but anisotropic, and describes a closed world. The solution differs in that it has "no horizon," i.e., the light signal has time to run around the entire world many times during the anisotropic stage.

During the early stage, matter (quanta, particles, antiparticles) does not influence the evolution of the solution. The gravitation of matter can be neglected.

If the matter has a homogeneous distribution, a transition to Friedmann's isotropic closed solution takes place in the course of time when the gravitation of the matter becomes appreciable.

The inhomogeneous distribution of matter can be regarded as a density perturbation superimposed on the homogeneous distribution. In the stage during which the gravitation of matter does not influence the general expansion, it should likewise not influence the perturbations. This means that there is no gravitational instability, and the perturbations attenuate, during the course of the adiabatic expansion, under the influence of the viscosity (particularly neutron viscosity [6]) and as the result of the formation of shock waves [7].

The initial inhomogeneity of the metric, which does not depend on the presence of matter ($R_{ik} = 0$), can be regarded as gravitational waves superimposed on the homogeneous solution. The gravitational waves also attenuate during the course of the expansion.

The "no horizon" condition is obviously necessary to permit equalization of the inhomogeneity and damping out of the perturbations. The closedness of the world limits the maximum length of the perturbation wave; this limitation is necessary for adiabatic damping to be applicable. It is the very possibility of equalization of the homogeneities which is a new property of the mixmaster universe. The anisotropic increase of the volume occurs in such a way that at each instant of time expansion takes place along two axes, and compression along the third. Perturbations with a momentum along the third axis increase adiabatically. However, owing to the bending of the trajectories and the interchange of the axis along which the compression takes place, one can assume that, on the average, the perturbations attenuate after a long time interval.

The attenuation of the anisotropy during the evolution of the anisotropic homogeneous solution and its transformation into an isotropic homogeneous solution was noted earlier, for example, for the Kasner solution [8].

We call attention in this paper to entropy perturbations. These are defined as perturbations of the ratio $s = \gamma/b$ of the density of the quanta $\gamma \sim T^3$ to the density of the baryan charge $b = B - \bar{B}$, where s is the dimensionless entropy, T is the temperature, B is the baryon density, and $\bar{B}$ is the antibaryon density.

The adiabatic perturbations ($\delta s = 0$, $\delta\rho/\rho = 3\delta T/T$ (they propagate with the velocity of sound $a = c/\sqrt{3}$, entropy perturbations become equalized only as a result of diffusion, i.e., quite slowly.

Under the conditions of the mixmaster universe, the entropy perturbations do not have time to equalize.

The stipulation of an initial state with perturbations of the density and the metric, but with s = const and $\delta s = 0$, seems artificial. But even with such an arbitrarily chosen initial state we get $\delta s \neq 0$ during the course of evolution, since the attenuation of the acoustic and gravitational waves is accompanied by a non-uniform heating of the matter, especially when shock waves are produced.

In the later stage, already after the isotropization of the over-all expansion of the universe and after recombination of the plasma, the entropy perturbations increase as the result of the gravitational instability; the singularities of this process are discussed in [9, 10].

We note that without allowance for the entropy perturbations, considering only adiabatic perturbations in the mixmaster universe, we would encounter the following difficulty: small-scale perturbations (of small wavelength) become equalized more strongly than perturbations of maximum scale. From this point of view, the absence of noticeable anisotropy of the relict radio emission cannot be easily reconciled with the large perturbations in the scale of galactic clusters.

Finally, in a mixmaster universe one cannot exclude the revival of the hypothesis of a cold initial state. In the homogeneous model, the anisotropic expansion is accompanied by an increase of the entropy[1)]. It is necessary that the present-day value of the entropy be attained before the nuclear reactions occur, i.e., at a temperature higher than 10^{10} deg. Only in this case do the calculations of the nuclear reactions in the Friedmann isotropic hot model, which give quite a reasonable composition of the primordial matter (70% H, 30% He^4), remain in force.

In the hot isotropic model, there are certain difficulties, namely, when $T \gtrsim Mc^2$ (M - proton mass) the concentrations of the baryons B and of the

[1)] A special calculation [11] shows that in the Friedmann isotropic model the increase of the entropy is small. In a flat isotropic model, the effects of the accumulation of entropy as the result of neutrino processes (and perhaps also graviton processes) are large [5, 12, 13]. The difference is formally explained by the fact that a relativistic gas has a zero second viscosity at a normal first viscosity.

antibaryons $\bar{B}$ are quite large compared with the baryon charge $b = B - \bar{B} \sim 10^{-8}(B + \bar{B})$. Such a composition calls for an explanation! For more details see [14].

In the cold model, it is assumed that during the initial stage there are only baryons everywhere, $\bar{B} = 0$, $b = B > 0$.

During the course of the evolution the temperature rises, owing to the anisotropic deformation, and many quanta, e^-, e^+ pairs, and even $B\bar{B}$ pairs are produced. However, an excess of baryons naturally remains, $b = B - \bar{B} > 0$, as is indeed observed [15]. From the point of view of the ordinary isotropic hot model with entropy perturbations, i.e., with $b \neq$ const, the absence of regions with $b < 0$ calls for a special explanation.

Let us note now some unsolved problems which were not touched upon above. During the earliest, quantum stages of the mixmaster model there is possible, in principle, a spontaneous production of particle-antiparticle pairs by a gravitational field, as the result of the rapid change of the metric [16].

It is necessary also to consider the behavior of the rotational and magnetic perturbations in the mixmaster model. The hypotheses advanced above require rigorous proof. Nevertheless, there is no doubt that we are at the beginning of a new stage in cosmology.

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FOCUSING OF LIGHT IN CUBIC MEDIA

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The interpretation of the presently accumulated large experimental material on the destruction of transparent media by focused laser radiation is greatly hindered by the absence of detailed numerical calculations of the picture of its self-focusing, similar to those performed for collimated beams [1 - 3]. Isolated numerical data, given in [4], do not change the overall picture, since they pertain only to two values of the focusing parameter $\nu = ka^2/F$.