point $\eta_{sf}^{-} < F$ remains[3].

Let us list also some applications of the transformation (2). First, it establishes the same rules for the construction of images in a lens for a cubic medium as for a linear medium (within the limits of the validity of Eq. (1)). The transformation (2) is applicable to nonstationary problems if ε^{NL} is a linear time functional of the field intensity (allowance for the relaxation time of the anisotropy, the nonstationary thermal self-focusing ($\Delta\varepsilon^{NL}$, $\int_0^t |E|^2 dt$), etc.)[4]. It is applicable also to problems of frequency mixing by a cubic nonlinearity (for example, to the effect of frequency tripling) in the case of linear synchronism. It follows from it, in particular, that the coefficient of frequency transformation at synchronism in the section from the lens to the focus (the lens is in contact with the nonlinear medium) does not depend on the focal length of the lens, and is equal to the coefficient of frequency conversion in a semi-infinite collimated beam. On the basis of the transformation (2) it is possible to analyze two-photon absorption and stimulated Raman scattering of focused beams.

In conclusion we note that the transformation (2) is not valid in the case of astigmatism of the lens, for example in cylindrical focusing (in the latter case a nonlinearity $\Delta\varepsilon^{NL}|E|^4$ is necessary). It can be used, however, in some cases of space-time focusing of radiation from frequency-modulated ribbon beams in dispersive cubic media.

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REGION OF FORMATION OF INTERNAL CONVERSION COEFFICIENTS ON HIGH SHELLS OF THE ATOM

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The formulas for the calculation of the internal conversion coefficients (ICC), assuming spherical symmetry of the field of the atom, in electric ($\lambda \equiv E$) and magnetic ($\lambda \equiv M$) transitions of a nucleus with multipolarity L, are

$$\alpha^{\lambda L} = \pi k \alpha \Sigma_\kappa B^{\lambda L}_{\kappa\kappa'} |R^{\lambda L}_{\kappa}|^2. \qquad (1)$$

Here k is the energy of the γ quantum, α is the fine structure, and $\kappa = 1/2 (\ell - j)(j + 1/2)$. The unprimed and primed indices pertain to electrons in the

[3] The absence of damage in transparent media behind a linear focus may be attributed to absorption of the power in the vicinity of the first convergence point.
[4] In the case of nonstationary striction self-focusing, the transformation (2) does not hold because of the dependence of the time of establishment of the nonlinearity on the dimensions of the beam cross section.

free state and to bound electrons, respectively. The $B_{\kappa\kappa}^{\lambda L}$ are calculated in final form by integrating over the angle variables. We consider radial integrals as functions of the upper limit r

$$R_{\kappa}^{ML}(r) = \int_0^r h_L(kr)(F_\kappa G_{\kappa'} + G_\kappa F_{\kappa'})\,dr \ , \tag{2}$$

$$R_{\kappa}^{EL}(r) = \int_0^r \{ h_{L-1}(kr)[(\kappa - \kappa')(G_\kappa F_{\kappa'} + F_\kappa G_{\kappa'}) + L(G_\kappa F_{\kappa'} - F_\kappa G_{\kappa'}] +$$

$$+ L h_L(kr)(F_\kappa F_{\kappa'} + G_\kappa G_{\kappa'})\}\,dr$$

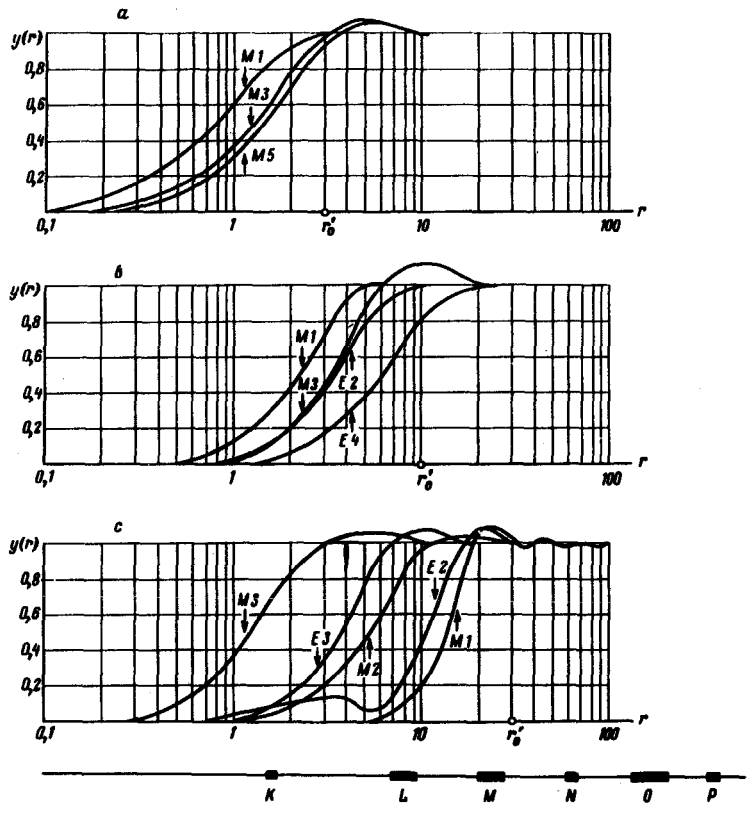
$h_L(kr)$ is the spherical Hankel function, and G_κ and F_κ are the "large" and "small" components of the relativistic radial wave function of the electron, multiplied by r. (We use the relativistic system of units).

It is customarily assumed that the radius of the region in which the ICC is formed is determined by the distances characteristic of the given process, namely 1/k, 1/p, and 1/p', which are connected with the behavior of the integrands in (2). ($p = \sqrt{\varepsilon^2 - 1}$, $p' = \sqrt{1 - \varepsilon'^2}$, ε and ε' are the energies of the free and the bound electrons). It followed from such considerations that at a small momentum of the conversion electron and at large N' - the principal number of the bound electrons - the ICC should be formed on the periphery of the atom. However, such estimates are incorrect at low energies of the conversion electron. The electron wave functions G_κ and F_κ are solutions of the Dirac radial equations

$$\left.\begin{aligned} dG_\kappa/dr &= -(\kappa/r)G_\kappa + (\varepsilon + 1 + \alpha Z\phi(r)/r)F_\kappa \\ dF_\kappa/dr &= (\kappa/r)F_\kappa - (\varepsilon - 1 + \alpha Z\phi(r)/r)G_\kappa \end{aligned}\right\} \tag{3}$$

($\phi(r)$ is the screening function), and the behavior of the functions at values of ε close to 1 is determined mainly by the potential term $\alpha Z\phi/r$. As the result, the radial functions begin to alternate in sign at small distances (close to $(\ell + 1)(\ell + 2)/\alpha Z$) from the center of the atom. The locations of the first zeroes of G_κ and $G_{\kappa'}$ are practically independent of 1/p or 1/p'.

We have considered the function $\gamma(r) = \alpha^{\lambda L}(r)/\alpha^{\lambda L}(\infty)$ and were interested in distances r_{eff} from the center of the atom such that $\gamma(r_{eff})$ reached its unity asymptotic value. It was found that r_{eff} (i.e., the radius of the region in which the ICC are formed) is smaller than the distance from the center of the atom to the first zeroes r_0' and r_0 of the wave functions of the electrons in the bound and in the free states at the values of κ making the largest contributions to (1). We have observed this regularity and verified by means of concrete calculations for all the shells of the atoms with Z = 72, 30, and 18 and for a number of conversion-electron energies $\varepsilon \geq 1.001\ mc^2$. As a rule, $r_{eff} \lesssim r_0'$. The figure shows the $\gamma(r)$ calculated for Z = 72 and $\varepsilon = 1.01\ mc^2$. The function $\gamma(r)$ depends little (within (15%) on ε at $1.001mc^2 \leq \varepsilon \leq 1.1mc^2$, and therefore all the curves pertain in practice to any energy in this interval. Each of the curves pertains in case (a) to the K, L_I, M_I, N_I, O_I, and P_I shells, in case (b) to L_{II}, M_{II}, N_{II}, and O_{II}, and in the case (c) to M_V and N_V. The abscissa scale is logarithmic. On the lower scale are shown, for comparison, the positions of the principal maxima of the corresponding electrons. The symbol r_0' denotes the position of the first zero of the function $G_{\kappa'}$ of the electron in the bound state. The type of transition is indicated alongside the



curves. Figures a - c demonstrate that $r_{eff} \lesssim r_0'$, but figure c shows that the r_{eff} differ greatly from each other in the case of conversion of one and the same shell of the atom but in different nuclear transitions. This is connected with the position of the first zeroes of r_0 of the wave functions of the free electrons. In all the cases given in the figure, $r_{eff} \lesssim \min(r_0', r_0)$. Qualitative estimates of this phenomenon, which can be obtained by changing over to the nonrelativistic limit for R_K^{EL} and by using the approximate formula of V.B. Berestetskii [1] for $R_{K'}^{ML}$, agree with our numerical calculations and conclusions.

So far we have been interested in the ratios $\alpha^{\lambda L}(r)$ and $\alpha^{\lambda L}(\infty)$, and ascertained that they reach their asymptotic value deep inside the atom. This means that the ICC is formed in that region, accurate to some factors determined by the usual requirements of normalization of the radial wave functions. The electric field on the periphery of the atom influences the ICC only through the normalizing factors, the squares of which are proportional to the electron density at zero.

From the foregoing we can draw the following conclusions: 1) the value of ICC on all the shells of the atom, including the valence shell, is formed in the internal layers of the atoms and therefore, apart from the change of the electron density at zero, does not depend on various changes of the electric field on the periphery of the atom; 2) measurement of the ICC on high atomic shells, as well as on the internal shells, can be used to determine the spins and parities of the nuclear levels; 3) the ICC on high shells of the atom should change little with changing number of electrons on other shells and change appreciably when the total number of electrons changes in that shell from which the conversion takes place. That is to say, the change of ICC on high shells of the atom can be used to determine the chemical state of the atom. This idea

was advanced in earlier papers [2, 3], but only the present analysis explains its causes; 4) measurement of the ICC can be used to determine the density of the electrons at zero [2, 3].

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ESTIMATE OF THE CONTRIBUTION OF TERMS OF ORDER e^2G TO THE ANOMALOUS MAGNETIC MOMENT OF THE MUON

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The contribution of the virtual weak interactions to the anomalous magnetic moment of the muon (g - 2), in the theory with an intermediate W boson, was calculated in [1 - 8] in the first order in the weak-interaction constant G and in the zeroth order in the electro-magnetic-interaction constant e. The contribution turned out to be small, since there was no quadratic divergence.

The purpose of the present paper is to estimate the contribution of terms of order e^2G, which contain quadratic divergences. There are two types of such divergences: 1) Divergences occurring in the case of integration over the large momenta of the virtual W boson. For these divergences, the natural cutoff is $\Lambda_W^2 \sim G^{-1}$. The presence of such terms would lead to relatively large corrections to the magnetic moment of the muon. 2) Divergences arising in the integration over large momenta of the virtual γ quantum. For such divergences it is necessary to expect cutoff due to the electromagnetic interactions of the W bosons, i.e., at $\Lambda_{el}^2 \sim \mu^2$ (μ - mass of W boson, $\alpha = 1/137$). These divergences do not lead to large corrections to g - 2.

We shall show below that terms with divergences of the former type in the anomalous moment of the muon vanish rigorously, so that the cutoff limit in the quadratically divergent terms of (g - 2), of order e^2G, should be of the order of $\Lambda_{el}^2 \sim \mu^2/\alpha$. In the analysis of the divergences of the former type, it can be assumed that the momentum of the virtual photon is much smaller than the momentum of the virtual W boson ($k' \ll k$).

Altogether there are 22 diagrams for terms of order e^2G in the magnetic moment of the muon. In nine of these diagrams, the quadratically-divergent terms are either missing or vanish after renormalization of the mass or charge of the muon. By calculating the degree of divergences it is easy to verify that seven diagrams in which the virtual photon line begins and ends on the W-boson line (an example of such a diagram is shown in Fig. 1) make no quadratic contribution to g - 2 upon integration with respect to k, if it is assumed that $k' \ll k$.

Each of the remaining six diagrams (Figs. 2 - 4 and their crossing-symmetry diagrams) contains a quadratic divergence upon integration with respect to k if $k' \ll k$. We shall show that when $k' \ll k$ the terms that diverge quadratically in k calcel each other in these three diagrams.

To calculate correctly the diverging integrals, these integrals must be cut off in a gauge-invariant manner, i.e., it is necessary to satisfy the generalized Ward identity for the vertices of the interaction of the W meson