was advanced in earlier papers [2, 3], but only the present analysis explains its causes; 4) measurement of the ICC can be used to determine the density of the electrons at zero [2, 3].

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ESTIMATE OF THE CONTRIBUTION OF TERMS OF ORDER e^2G TO THE ANOMALOUS MAGNETIC MOMENT OF THE MUON

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The contribution of the virtual weak interactions to the anomalous magnetic moment of the muon (g - 2), in the theory with an intermediate W boson, was calculated in [1 - 8] in the first order in the weak-interaction constant G and in the zeroth order in the electro-magnetic-interaction constant e. The contribution turned out to be small, since there was no quadratic divergence.

The purpose of the present paper is to estimate the contribution of terms of order e^2G, which contain quadratic divergences. There are two types of such divergences: 1) Divergences occurring in the case of integration over the large momenta of the virtual W boson. For these divergences, the natural cutoff is $\Lambda_W^2 \sim G^{-1}$. The presence of such terms would lead to relatively large corrections to the magnetic moment of the muon. 2) Divergences arising in the integration over large momenta of the virtual γ quantum. For such divergences it is necessary to expect cutoff due to the electromagnetic interactions of the W bosons, i.e., at $\Lambda_{el}^2 \sim \mu^2$ (μ - mass of W boson, α = 1/137). These divergences do not lead to large corrections to g - 2.

We shall show below that terms with divergences of the former type in the anomalous moment of the muon vanish rigorously, so that the cutoff limit in the quadratically divergent terms of (g - 2), of order e^2G, should be of the order of $\Lambda_{el}^2 \sim \mu^2/\alpha$. In the analysis of the divergences of the former type, it can be assumed that the momentum of the virtual photon is much smaller than the momentum of the virtual W boson (k' << k).

Altogether there are 22 diagrams for terms of order e^2G in the magnetic moment of the muon. In nine of these diagrams, the quadratically-divergent terms are either missing or vanish after renormalization of the mass or charge of the muon. By calculating the degree of divergences it is easy to verify that seven diagrams in which the virtual photon line begins and ends on the W-boson line (an example of such a diagram is shown in Fig. 1) make no quadratic contribution to g - 2 upon integration with respect to k, if it is assumed that k' << k.

Each of the remaining six diagrams (Figs. 2 - 4 and their crossing-symmetry diagrams) contains a quadratic divergence upon integration with respect to k if k' << k. We shall show that when k' << k the terms that diverge quadratically in k calcel each other in these three diagrams.

To calculate correctly the diverging integrals, these integrals must be cut off in a gauge-invariant manner, i.e., it is necessary to satisfy the generalized Ward identity for the vertices of the interaction of the W meson

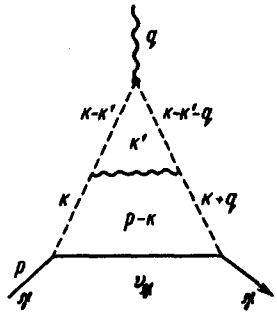

Fig. 1

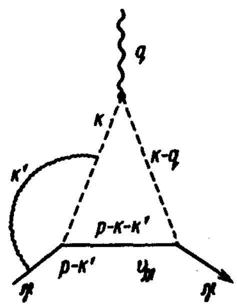

Fig. 2

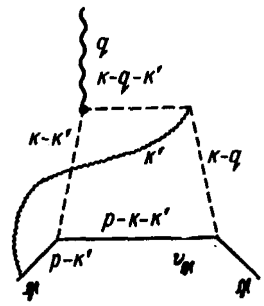

Fig. 3

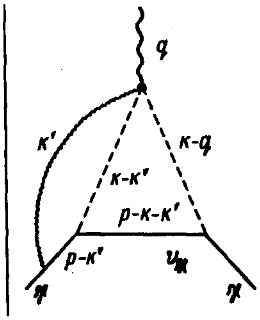

Fig. 4

with the electromagnetic field $\Gamma^{\lambda}_{\nu\mu}$ and $U^{\lambda\rho}_{\nu\mu}$. At small γ-quantum momenta $q << k$ where k is the W-meson 4-momentum), it follows from the generalized Ward identities the following form for the vertices $\Gamma^{\lambda}_{\nu\mu}$ and $U^{\lambda\rho}_{\nu\mu}$, accurate to terms linear in q:

$$G_{\nu\mu}(k) = \frac{1}{k^2 - \mu^2}\left(\delta_{\nu\mu} - \frac{k_\nu k_\mu}{\mu^2}\right)\left(\frac{\Lambda^2}{\Lambda^2 - k^2}\right)^2, \tag{1}$$

$$\Gamma^{\lambda}_{\nu\mu}(k + q/2,\ k - q/2) = \frac{\partial G^{-1}_{\nu\mu}(k)}{\partial k_\lambda} + \left(\frac{1}{2} + \gamma\right)\left(\frac{\Lambda^2 - k^2}{\Lambda^2}\right)^2 (q_\mu \delta_{\nu\lambda} - q_\nu \delta_{\mu\lambda}). \tag{2}$$

$$U^{\lambda\rho}_{\nu\mu}(k,\ k',\ q) = \frac{\partial \Gamma^{\lambda}_{\nu\mu}\left(k + \frac{k'}{2},\ k - \frac{k'}{2}\right)}{\partial k_\rho} = \frac{\partial \Gamma^{\rho}_{\nu\mu}(k + q/2,\ k - q/2)}{\partial k_\lambda}, \tag{3}$$

where γ is the anomalous magnetic moment of the W meson. We note first that the second term in the right side of (2), which is connected with the magnetic moment of the W meson, yields no quadratic divergences of the first type, and will therefore be disregarded.

In diagrams 2 - 4, the quadratic divergence arises upon integration with respect to k only because of the terms of zeroth and first order in the expansions in powers of k'/k and q'/k, so that the contribution of these diagrams should take the form

$$\int \frac{d^4k'}{k'^2} \int d^4k \{A(k) + B(k)k' + C(k)q\}_{\lambda\nu\mu\rho}\, \bar{u}\gamma_\nu(1+\gamma_5)(\hat{p} - \hat{k} - \hat{k}')^{-1} \times$$

$$\times\, \gamma_\mu(1+\gamma_5)\,\gamma_\rho(\hat{p} - \hat{k}' - m)^{-1} u\ , \tag{4}$$

where A, B, and C are functions of k only. The contribution of the terms A(k) + B(k)k' in (2) can be calculated by putting q = 0 in the curly brackets of (4) and using formulas (2) and (3)

$$-\int \frac{d^4k'}{k'^2} \int d^4k \frac{d}{\partial k_\zeta} \{ G_{\nu\sigma}(k) \Gamma^{\lambda}_{\sigma\rho}(k, k-k') G_{\rho\mu}(k-k') \} \bar{u} \gamma_\nu (1+\gamma_5) \times$$

$$\times (\hat{p} - \hat{k} - \hat{k}')^{-1} \gamma_\mu (1+\gamma_5) \gamma_\zeta (\hat{p} - \hat{k}' - m)^{-1} u .$$

It is easy to see that (4) does not contain a quadratic divergence following integration with respect to k. Analogously, setting k' = 0 and using Ward's identity at the vertex of the interaction of the W boson with the virtual photon, it is easy to show that A(k) + C(k)q yields no quadratic divergence in k. By the same token, we show that when $k' \ll k$ the anomalous magnetic moment of the muon has no terms that diverge quadratically in k in the order e^2G, and consequently these terms result only from the momenta of the virtual photons k', of the order of the W-boson momenta. In this case it is nautral to assume that the cutoff is due to the electromagnetic interaction of the W bosons, i.e., the cutoff limit is of the order of $\sim\Lambda^2_{el} \sim \mu^2/\alpha$.

We present in conclusion an estimate of the contribution of terms of order e^2G to the quantity (g - 2)/2 for the muon (at $\gamma = 0$)

$$(g-2)/2 \sim (4\pi e^2)(4\pi g^2) \frac{\pi^4}{(2\pi)^8} \frac{\Lambda^2 m^2}{\mu^4} \ln \frac{\Lambda^2}{m^2} \sim \frac{\alpha}{(4\pi)^3} \frac{G\Lambda^2 m^2}{\mu^2} \ln \frac{\Lambda^2}{m^2} \qquad (5)$$

(m - muon mass). At $\Lambda^2 = \Lambda^2_{el} \sim \mu^2/\alpha$, it follows from this that

$$\frac{g-2}{2} \sim \frac{1}{(4\pi)^3} G m^2 \ln \frac{\Lambda^2}{m^2} \sim 10^{-9},$$

which is smaller by two orders of magnitude than the experimental accuracy attained at the present time.

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AMPLITUDE OF SCATTERING OF A NON-VACUUM REGGEON BY A PARTICLE

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We have calculated in this paper, using the Veneziano model, the amplitude for the scattering of a non-vacuum reggeon by a particle; this amplitude is needed for the description of the contribution of two-reggeon branch cuts. The reggeon-particle scattering amplitude obtained by factorization of the six-point diagram, at a certain choice of the asymptotic regime, contains three terms similar in meaning to the three terms of the usual Veneziano four-point diagram. The two-reggeon branch cuts were calculated without allowance for the "enhanced" diagrams [1].