$$- \int \frac{d^4k'}{k'^2} \int d^4k \frac{d}{\partial k_\zeta} \{ G_{\nu\sigma}(k)\Gamma^\lambda_{\sigma\rho}(k, k-k') G_{\rho\mu}(k-k') \} \bar{u}\gamma_\nu(1+\gamma_5) \times$$

$$\times (\hat{p} - \hat{k} - \hat{k}')^{-1} \gamma_\mu (1+\gamma_5) \gamma_\zeta (\hat{p} - \hat{k}' - m)^{-1} u .$$

It is easy to see that (4) does not contain a quadratic divergence following integration with respect to k. Analogously, setting k' = 0 and using Ward's identity at the vertex of the interaction of the W boson with the virtual photon, it is easy to show that A(k) + C(k)q yields no quadratic divergence in k. By the same token, we show that when k' << k the anomalous magnetic moment of the muon has no terms that diverge quadratically in k in the order e^2G, and consequently these terms result only from the momenta of the virtual photons k', of the order of the W-boson momenta. In this case it is nautral to assume that the cutoff is due to the electromagnetic interaction of the W bosons, i.e., the cutoff limit is of the order of $\sim\Lambda^2_{el} \sim \mu^2/\alpha$.

We present in conclusion an estimate of the contribution of terms of order e^2G to the quantity (g - 2)/2 for the muon (at γ = 0)

$$(g-2)/2 \sim (4\pi e^2)(4\pi g^2) \frac{\pi^4}{(2\pi)^8} \frac{\Lambda^2 m^2}{\mu^4} \ln \frac{\Lambda^2}{m^2} \sim \frac{\alpha}{(4\pi)^3} \frac{G\Lambda^2 m^2}{\mu^2} \ln \frac{\Lambda^2}{m^2} \qquad (5)$$

(m - muon mass). At $\Lambda^2 = \Lambda^2_{el} \sim \mu^2/\alpha$, it follows from this that

$$\frac{g-2}{2} \sim \frac{1}{(4\pi)^3} Gm^2 \ln \frac{\Lambda^2}{m^2} \sim 10^{-9},$$

which is smaller by two orders of magnitude than the experimental accuracy attained at the present time.

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AMPLITUDE OF SCATTERING OF A NON-VACUUM REGGEON BY A PARTICLE

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Submitted 12 February 1970
ZhETF Pis. Red. 11, No. 6, 312 - 316 (20 March 1970)

We have calculated in this paper, using the Veneziano model, the amplitude for the scattering of a non-vacuum reggeon by a particle; this amplitude is needed for the description of the contribution of two-reggeon branch cuts. The reggeon-particle scattering amplitude obtained by factorization of the six-point diagram, at a certain choice of the asymptotic regime, contains three terms similar in meaning to the three terms of the usual Veneziano four-point diagram. The two-reggeon branch cuts were calculated without allowance for the "enhanced" diagrams [1].

Let us examine a six-point diagram for scalar particles of mass M. In the generalized Veneziano theory, it is described by sixty nonequivalent diagrams, one of which is shown in Fig. 1, and the remainder differ from it in permutation of the particles [2]. In the asymptotic regime corresponding to the transition from Fig. 1a to Fig. 1b, twelve out of these 60 diagrams are most significant (as will be discussed more concretely below). Their amplitudes factor out, and the contributions corresponding to the diagrams in Fig. 2 to the reggeon-particle scattering amplitude are separated. The expressions describing them are similar to one another; we shall therefore describe in relative detail only the contribution of the diagram of Fig. 2a, which corresponds to Figs. 1a and b.

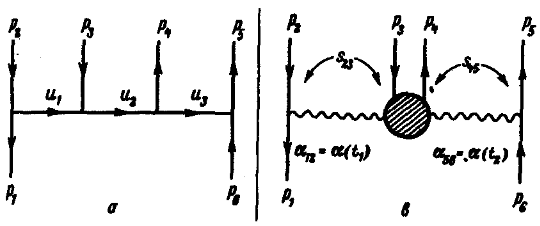

Fig. 1

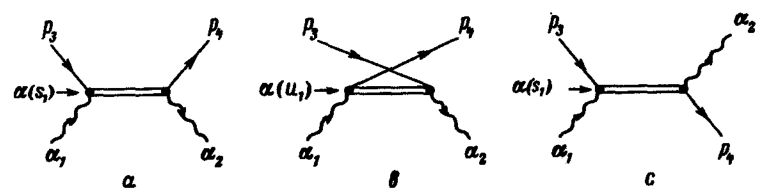

Fig. 2

To describe the six-point diagram, we use the following set of 9 variables (among which only 8 are independent):

$$\begin{aligned} &s_{12}=t_1=(p_2-p_1)^2, && s_{45}=(p_4+p_5)^2, && s_{123}=s_1=(p_1-p_2-p_3)^2,\\ &s_{23}=(p_2+p_3)^2, && s_{56}=t_2=(p_6-p_5)^2, && s_{234}=(p_2+p_3-p_4)^2,\\ &s_{34}=t=(p_4-p_3)^2, && s_{16}=(p_6-p_1)^2, && s_{345}=(p_4+p_5-p_3)^2. \end{aligned} \tag{1}$$

The amplitude of the six-point diagram was calculated in the Fubini-Gordon-Veneziano operator formalism [3]. In the asymptotic regime s_{23}, $s_{45}\to\infty$, and the contribution of the diagram of Fig. 1 is described by the following espression:

$$\begin{aligned} B_6(1) = &\int_0^{(1)} du_1 \int_{(0)}^{1} du_2 \int_0^{(1)} du_3\, u_1^{-\alpha_{12}-1} u_2^{-\alpha_{123}-1} u_3^{-\alpha_{56}-1} [(1-u_1)(1-u_2)(1-u_3)]^{\alpha_0-1} \times \\ &\times \exp\{2\alpha'[(p_2p_3)u_1-(p_2p_4)u_1u_2-(p_3p_4)\sum_{n=1}^{\infty}\frac{u_2^n}{n}- \\ &-(p_2p_5)u_1u_2u_3-(p_3p_5)u_2u_3+(p_4p_5)u_3]\}, \\ &\mathrm{Re}\,\alpha_{12},\ \mathrm{Re}\,\alpha_{56}<0,\quad \mathrm{Re}\,\alpha_{123}>0, \end{aligned} \tag{2}$$

where $\alpha_i = \alpha(s_i) = \alpha_0 + \alpha' s_i$ is the Regge trajectory to which the scalar partixles considered above belong. When s_{23}, $s_{45}\to\infty$, s_{12}, s_{56}, $s_{34} \ll s_{23}$, s_{45}, and s_{123}, s_{234}, and s_{345} are arbitrary, we have

$$\begin{aligned} B_6(1) = &(-\alpha' s_{23})^{\alpha_1}(-\alpha' s_{45})^{\alpha_2}\Gamma(-\alpha_1)\Gamma(-\alpha_2)\int_{(0)}^{1} du\, u^{-\alpha(s_1)-1}(1-u)^{-\alpha(t)-1}\times \\ &\times c_1^{\alpha_1}c_2^{\alpha_2}\, {}_2F_0\Big(-\alpha_1,-\alpha_2;-\frac{u}{\alpha'\eta c_1 c_2}\Big) = (-\alpha' s_{23})^{\alpha_1}(-\alpha' s_{45})^{\alpha_2}\Gamma(-\alpha_1)\Gamma(-\alpha_2)A^{(a)}_{\alpha_1\alpha_2}\times \\ &\times (s_1, t, t_1, t_2, \mu_1, \mu_2), \end{aligned} \tag{3}$$

where

$$c_{1,2}=1+\left(\frac{1}{\mu_{1,2}}-1\right)v, \quad \alpha_{1,2}=\alpha(t_{1,2}), \quad \mu_1=\frac{s_{23}}{s_{234}}, \quad \mu_2=\frac{s_{45}}{s_{345}},$$

$$\eta=-\frac{s_{23}s_{45}}{s_{16}},$$

${}_2F_0(\alpha, \beta; x)$ is the confluent hypergeometric function. $A^{(a)}_{\alpha_1\alpha_2}$ is the contribution to the diagram of Fig. 2a to the amplitude for the scattering of a reggeon by a particle. The quantities μ_1, μ_2, and η are functions of the variables characterizing the amplitude $A_{\alpha_1\alpha_2}$. In particular, the functions $\mu_{1,2}$ depend on s_1, t, t_1, t_2, and the azimuthal angles connected with the helicities of the reggeons. The function $\eta(s_1, t, t_1, t_2, \mu_1, \mu_2)$ is determined by the nonlinear relation between the nine Mandelstam variables (1) [4].

Expanding (3) in terms of the poles of the variable $\alpha(s_1)$, we can easily verify that the contribution of $A^{(a)}_{\alpha_1\alpha_2}$ is an infinite sum over the resonances in the s-channel of the reggeon-particle scattering (Fig. 2a). The aforementioned 12 significant diagrams of the six-point diagram corresponds to the following particle permutations:

$$\begin{array}{lll}
(123456)\to B_6(1) & (124356)\to B_6(5) & (123654)\to B_6(9), \\
(213456)\to B_6(2) & (214356)\to B_6(6) & (213654)\to B_6(10), \\
(123465)\to B_6(3) & (124365)\to B_6(7) & (123564)\to B_6(11), \\
(213465)\to B_6(4) & (214365)\to B_6(8) & (213564)\to B_6(12).
\end{array}$$

Simultaneous examination of diagrams $B_6(1)$ - $B_6(4)$ leads to the appearance in formula (3) of the signature factors of the reggeons α_1 and α_2. By factoring the amplitudes $B_6(5)$ - $B_6(8)$ and $B_6(9)$ - $B_6(12)$ we can separate the contributions of the diagrams of Figs. 2b and 2c, respectively. The total reggeon-particle scattering amplitude is given by

$$A_{\alpha_1\alpha_2}(s_1, t, t_1, t_2, \mu_1, \mu_2)=A^{(a)}_{\alpha_1\alpha_2}+A^{(b)}_{\alpha_1\alpha_2}+A^{(c)}_{\alpha_1\alpha_2}=$$

$$=A^{(a)}_{\alpha_1\alpha_2}\left(\alpha(s_1), \alpha(t), 1-\frac{1}{\mu_{1,2}}\right)+A^{(a)}_{\alpha_1\alpha_2}\left(\alpha(u_1), \alpha(t), \frac{\mu_{1,2}}{\mu_{1,2}-1}\right)+$$

$$+A^{(a)}_{\alpha_1\alpha_2}\left(\alpha(u_1), \alpha(s_1), \frac{1}{\mu_{1,2}}\right), \tag{4}$$

where

$$u_1=s_{124}=2M^2+t_1+t_2-t-s_1.$$

When $\alpha_1 = \alpha_2 = 0$, formula (4) goes over to the well-known Veneziano formula for a four-point diagram of scalar particles [4].

Formulas (3) and (4) make it possible to describe the total contribution of a sequence of resonances R_1 and R_2 to the amplitude of double rescattering $f(s, t)$ (Fig. 3). In accordance with the definition (3), we have (see [6 - 8]):

$$f(s,t)=-\frac{1}{4}\int\frac{d^2k_\perp}{(2\pi)^2}N^2(t, t_1, t_2)\frac{\sigma_{\alpha_1}\sigma_{\alpha_2}}{\Gamma(1+\alpha_1)\Gamma(1+\alpha_2)}(\alpha's)^{\alpha_1+\alpha_2}, \tag{5}$$

where σ_α is the signature factor.

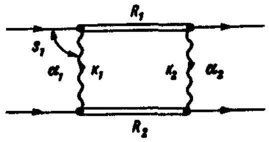

Fig. 3

Landshoff [9] has shown that the function $N(t,t_1,t_2)$, defined by the integral of th reggeon-particle scattering amplitude with respect to s_1 [7, 8] is described by the amplitude (4) only in the limit as $\mu_1,\ \mu_2 \to \infty$:

$$N(t,t_1,t_2) = \alpha' g^2 \int_{-\infty}^{\infty} \frac{ds_1}{2\pi i} \lim_{\mu_1,\mu_2\to\infty} A_{\alpha_1\alpha_2}(s_1,t,t_1,t_2,\mu_1,\mu_2), \qquad (6)$$

where g is the coupling constant of three scalar particles. The transition to the limit $\mu_{1,2} \to \infty$ (and at the same time $\eta \to \infty$) is equivalent to the summation over the reggeon helicities, which is necessary to calculate the loop of Fig. 3 [9]. According to (3)

$$\lim_{\mu_{1,2}\to\infty} A_{\alpha_1\alpha_2} = B(-\alpha(s_1),-\alpha(t)+\alpha_1+\alpha_2) + B(-\alpha(u_1),-\alpha(t)+\alpha_1+\alpha_2) + B(-\alpha(s_1),-\alpha(u_1)). \qquad (7)$$

The integral (6) is meaningful only if the amplitude

$$\lim_{\mu_{1,2}\to\infty} A_{\alpha_1\alpha_2}$$

decreases sufficiently rapidly with increasing s_1 When $\mathrm{Re}\,\alpha(s_1)$, $\mathrm{Im}\,\alpha(s_1) \to \infty$ $(\mathrm{Im}\,\alpha(s_1)/\mathrm{Re}\,\alpha(s_1) \ll 1)$ we have

$$\lim_{\mu_{1,2}\to\infty} A^{\mathrm{Reg}}_{\alpha_1\alpha_2} = \lim_{s_1,\mu_{1,2}\to\infty} [A^{(a)}_{\alpha_1\alpha_2} + A^{(b)}_{\alpha_1\alpha_2}] = \frac{\pi\sigma_{\alpha(t)} - \alpha_1 - \alpha_2}{\Gamma(1+\alpha(t)-\alpha_1-\alpha_2)} s_1^{\alpha(t)-\alpha_1-\alpha_2}. \qquad (8)$$

Allowance for the contribution of (8) is connected with the "enhanced" diagrams and calls for separate analysis [10]. Substituting in the integral (6) the difference $A_{\alpha_1\alpha_2} - A^{\mathrm{Reg}}_{\alpha_1\alpha_2}$, we can readily show that

$$N(t,t_1,t_2) = \alpha' g^2 \int_{-\infty}^{\infty} \frac{ds_1}{2\pi i} \lim_{\mu_{1,2}\to\infty} A^{(c)}_{\alpha_1\alpha_2} =$$

$$= g^2 \Sigma\text{выч}\, B(-\alpha(s_1),-\alpha(u_1)) \Big|^{-\infty}_{\alpha(s_1)=\ell} = g^2 \sum_{\ell=0}^{\infty} (-1)^{\ell} \frac{\Gamma[\ell+\alpha'(t-t_1-t_2)]}{\Gamma[\alpha'(t-t_1-t_2)]} =$$

$$= g^2 \exp[\alpha'(t_1+t_2-t)\ln 2]. \qquad (9)$$

The author is deeply grateful to O.V. Kancheli, S.G. Matinyan, and K.A. Ter-Martirosyan, for useful discussion and valuable device, and to I.D. Mandzhavidze and Dzh.L. Chkarueli for interest in the work.

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TURBULENCE IN AN ISOTROPIC UNIVERSE

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Submitted 25 November 1969; resubmitted 18 February 1970
ZhETF Pis. Red. 11, No. 6, 317 - 319 (20 March 1970)

A number of observational and theoretical arguments offer definite evidence in favor of the hypothesis [1, 2] that a developed turbulence exists in the intergalactic medium during the epoch of galaxy formation at an average density $\rho = \rho_G \simeq 10^{-23} - 10^{-25}$ g/cm^3 and at a universe age $t = t_G \simeq 10^{15} - 10^{16}$ sec. Direct extrapolation of the notions of metagalactic turbulence to the past ($t < t_G$) leads to the conclusion of an anisotropic, non-Friedmann character of the earlier phase of cosmological expansion [3, 4], which is equivalent to a "primordial" existence of strong turbulent vortical motions that determine the metric properties of space-time. However, an analysis of the hydrodynamics of a "hot" universe under conditions when the radiation density ρ_r exceeds the density of matter ρ gives grounds for a fundamentally different solution of the problem of the earlier expansion phase. We shall show that the state of developed metagalactic turbulence need not necessarily be "primordial"; it could arise during the period of recombination of the cosmic plasma, owing to small (in a sense indicated below) motions of a potential or an acoustic type. These motions, unlike vortical motions, are compatible with the isotropy of the metric in all the phases of expansion.

The evolution of potential motions superimposed on the regular cosmological expansion consists of the following three stages: 1) gravitational instability, in which the motion velocity $v \sim t^{3/2}$ increases while the perturbations of the metric remain constant in time, and the characteristic scale of motion ℓ exceeds the critical Jeans length $\ell_1 \simeq u(G\rho)^{-1/2}$ (here u is the speed of sound, $u = u_0 = c/\sqrt{3}$ when $\rho_r > \rho$); 2) conversion of the motions into acoustic waves of constant amplitude, corresponding to time-decreasing perturbations of the metric and to a wavelength ℓ smaller than ℓ_1; 3) hydrodynamic instability.

The transition from stage 1) to stage 2) is determined by the fact that the ratio $\ell/\ell_1 \sim t^{-1/2}$ decreases during the course of the expansion. According to the general theory [5], these motions produce either a small perturbation in an isotropic universe (i.e., the relative deviation of the metric from the Friedmann metric is always smaller than unity), if in phase 2) the amplitude of the velocity is not too close to the velocity of sound u_0:

$$v < (1/10)u_0 \approx 2 \cdot 10^9 \text{ cm/sec.} \qquad (1)$$

Phase 3) is due to physical phenomena in a "hot" universe, not accounted for by the theory [5] and its cosmogonic applications [6 - 8]. The hydrodynamic instability arises during the time of a sharp decrease of the velocity of sound