We note that none of the foregoing contradicts the general theorems concerning the connection between flux quantization and the presence of superconducting long-range order [3]. Formally, this effect is not a macroscopic quantum effect, since in a large system (as $R \to \infty$) the magnetic moment μ vanishes (see (6) and (7)). It is more readily analogous to the diamagnetism of closed organic molecules. However, as shown by the foregoing estimates, when certain conditions are satisfied, the effect can appear in samples of rather large dimensions, usually regarded as "macroscopic."

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NONLINEAR WAVES IN A RELATIVISTIC ELECTRON BEAM

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It is known that in stationary electronic configuration (the self-stabilized Budker beam [1], Veksler rings [2]), a decrease of the Coulomb repulsion pulses is attained with the aid of the Lorentz forces due to the contraction of the currents of relativistic electrons. Since the limitation of the wave amplitude in a plasma is connected with the action of Coulomb repulsion of the electrons, the natural question arises whether it is possible to excite large-amplitude currents by enhancing the foregoing compensation. Such a situation obtains when wave propagate along the axis of an electron beam whose particles rotate azimuthally (a system of the E-layer type [3]). In this case the charge-density wave leads to oscillations of the particle current in the beam $j'_\phi(z - v_{ph}t) = -ev_0 n'(z - v_{ph}t)$ (v_0 is the azimuthal velocity of the beam), and consequently to the appearance of the magnetic field of the wave, $H_r(z - v_{ph}t)$. the self-contraction force, produced by this magnetic field in the plasmoids into which the wave breaks up the beam, $F_H = ev_0H_r/c$, just as in the stationary case, is in anti-phase with the Coulomb force $F_E = -eE_z$, and leads at $v_0 \approx (c^2 - v_{ph}^2)^{1/2}$ to an appreciable decrease of the electron displacement in the field of the wave. A wave can then propagate in the beam, having a very large electric-field amplitude, without intersection of the trajectories and breaking of the wave front. This result points to the possibility of effectively using waves in relativistic beams to implement the plasma method of acceleration proposed in [4].

For simplicity, we consider in the present paper the case of rectangular geometry, namely an electron beam moves along the y axis and is bounded in x. The electron charge of the beam is assumed to be partially compensated by the ions and at equilibrium the Coulomb repulsion force acting on the electrons, $-eE_x$, is balanced by the magnetic force ev_0H_z/c, where $H_z(x)$ is the magnetic field produced by the electron current. Equilibrium solutions of this type were considered in [1, 5].

The wave produced in such an electron beam can be described with the aid of the usual hydrodynamic system of equations. All the wave quantities in these equations depend on $\xi = z - v_{ph}t$, and the transverse gradients of these quantities are small compared with the longitudinal ones.

$$\left|\frac{d\ln f}{d\xi}\right| \Big/ \left|\frac{d\ln f}{dx}\right| \sim ka >> 1 \tag{1}$$

(a - transverse beam dimension, k - wave number).

The equations of motion then have the following energy integral

$$\mathcal{E} - v_{ph}p_z = \mathcal{E}_0 + v_0(p_y - p_0) + e\phi - \frac{e}{c} v_0 A_y . \tag{2}$$

In this equation $A(\xi)$ and $\phi(\xi)$ are the vector and scalar potentials of the wave, $\mathcal{E} = (m^2c^4 + c^2p^2)^{1/2}$ is the electron energy, and $\mathcal{E}_0 = (m^2c^4 + c^2p_0^2)^{1/2}$ is the equilibrium value of the energy at the point $\phi = A_y = 0$.

The proper magnetic field $H_z(x)$ of the beam current magnetizes the transverse wave motions in the beam, since the cyclotron part of the electrons in this field is $\omega_{H_z} = \omega_0^2 a/c >> \omega_0$, and the frequency of the investigated oscillations is $\omega \lesssim \omega_0$ ($\omega_0 = (4\pi e^2 n_0/m\gamma_0)^{1/2}$ is the Langmuir frequency and $\gamma_0 = \mathcal{E}/mc^2$). Under these conditions

$$v_x = p_x = 0, \quad v_y = v_0 ,$$

$$p_y = \frac{v_0}{c^2} \mathcal{E} = p_0 \left[1 + \frac{e\left(\phi - \frac{v_0}{c} A_y\right) + p_z v_{ph}}{\mathcal{E}_0} \gamma_0^2 \right], \tag{3}$$

and the equation for the potential $\phi(\xi)$ takes the form

$$\frac{d^2\phi}{d\xi^2} = 4\pi e n_0 \frac{c^2}{c^2 - v_{ph}^2\gamma_0^2} \left[\frac{v_{ph}\gamma_0 w}{\sqrt{w^2c^2 - m^2c^4(c^2 - v_{ph}^2\gamma_0^2)}} - 1 \right], \tag{4}$$

где $w = mc^2 + e\phi \dfrac{c^2 - v_{ph}^2\gamma_0^2}{\gamma_0 (c^2 - v_{ph}^2)}$.

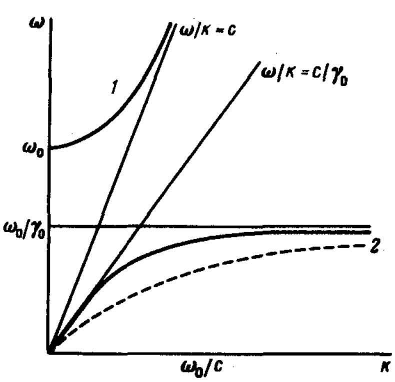
The solution (4) leads to the following nonlinear dispersion equation for the wave under consideration:

$$k^2 v_{ph}^2 = \omega_0^2 \frac{c^2 - v_{ph}^2\gamma_0^2}{(c^2 - v_{ph}^2)\gamma_0^2} \frac{1}{\lambda + \sqrt{\lambda^2 - 1}} \left(\frac{\pi}{2E(\kappa)}\right)^2 . \tag{5}$$

The wave number is $k = 2\pi/\xi_0$, where ξ_0 is the period of $\phi(\xi)$,

$$\lambda = 1 + \frac{E^2_{max}}{4\pi n_0 \mathcal{E}_0} \quad \frac{c^2 - v_{ph}^2\gamma_0^2}{\gamma_0^2(c^2 - v_{ph}^2)} ,$$

E_{max} is the amplitude of the longitudinal field in the wave $E_z(\xi)$, $\kappa^2 = 2(\lambda^2 - 1)^{1/2}/\lambda + (\lambda^2 - 1)^{1/2}$, and $E(\kappa)$ is a complete elliptical integral of the second kind. It follows from (5) that two types of waves exist in the electron beam (see the figure), fast waves with $v_{ph} > c$ (curve 1) and slow waves with a phase velocity in the range $0 < v_{ph} < c/\gamma_0 = (c^2 - v_0^2)^{1/2}$ (curve 2). With increasing amplitude of the slow wave, its frequency at a given k decreases, but the limits within which the phase velocity of this wave varies and the maximum value of the frequency $\omega_{max} = \omega_0/\gamma_0$ do not change. The slow wave has appreciable components along both the longitudinal (E^{ℓ}) and the transverse (E^{tr}) electric field, and the connection between these components at $v_{ph} \simeq c/\gamma_0$ is determined by the relation



Dispersion relations ω(k) for waves in an azimuthal electron beam; solid curves - small amplitudes ($\lambda \to 1$), dashed - maximum amplitudes as determined by Eq. (7)

$$E^{\ell} = - \frac{v_0}{v_{ph}} E^{tr} . \tag{6}$$

The magnetic force acting in the longitudinal direction on the beam electrons

$$F_H = \frac{e}{c} v_0 H_x = - \frac{e}{c} v_0 \frac{dA_y}{d\xi} = - e \frac{v_0^2}{c^2 - v_{ph}^2} \frac{d\phi}{d\xi}$$

compensates in the case when $v_{ph} \to (c^2 - v_0^2)^{1/2}$ the Coulomb force $F_E = e(d\phi/d\xi)$. Under these conditions one can expect an appreciable increase of the maximum possible value of the wave amplitude.

Indeed from Eq. (4) it is possible to obtain in the usual manner the following formula for the maximum amplitude of the longitudinal electric field, at which "breaking" of the wave front takes place, $E_{max\ max}$:

$$E^2_{max\ max} = 8\pi n_0 \mathcal{E}_0 \left(1 - \frac{v_{ph}^2}{c^2}\right) \frac{1 - \left(1 - \frac{v_{ph}^2\gamma_0^2}{c^2}\right)^{1/2}}{\left(1 - \frac{v_{ph}^2\gamma_0^2}{c^2}\right)^{3/2}} . \tag{7}$$

$$\left(0 < v_{ph} < \frac{c}{\gamma_0}\right) .$$

When $\omega \simeq (\omega_0/\gamma_0)|v_{ph} << c/\gamma_0|$, the amplitude $E_{max\ max}$ is sufficiently small

$$\frac{E^2_{max\ max}}{8\pi} \simeq n_0 m v_{ph}^2 \gamma_0^3 << n_0 \mathcal{E}_0 . \tag{8}$$

When v_{ph} approaches c/γ_0, the amplitude $E_{max\ max}$ increases rapidly, but remains bounded, since by virtue of condition (1) v_{ph} cannot be arbitrarily close to c/γ_0. Using this condition and the dispersion equation (5) we can obtain an estimated upper bound for $E_{max\ max}$:

$$\frac{E^2_{max\ max}}{8\pi} \sim n_0 \mathcal{E}_0 \frac{\omega_0^2 a^2}{c^2}. \tag{9}$$

Since $\omega_0 a/c >> 1$, the maximum energy of the electric field of the wave greatly exceeds the beam energy.

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POSSIBLE VERIFICATION OF THE POMERANCHUK THEOREM IN Kd INTERACTIONS

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According to recent experiments performed at Serpukhov [1], there is an unexpectedly large difference between the total cross sections of K^+ and K^- mesons on protons and deuterons. This situation casts doubts on the validity of the Pomeranchuk theorem [2]. Since the dispersion relations (d. r.) impose strong limitations on the energy dependence of the real and imaginary parts of the scattering amplitudes, it is possible to use the experimental information on the phase shifts of the forward amplitudes at the energies attainable in contemporary accelerators, in order to clarify the character of the behavior of the total cross sections at higher energies.

In [3] we used d.r. to predict the phase shifts of the amplitudes of $K^\pm p$ and $K^\pm n$ scattering and the amplitude of K^0-meson regeneration on protons, assuming that the approximately-constant values of the total cross sections, measured in Serpukhov [1], constitute the asymptotic values. Similar reasoning for $K^\pm d$ scattering makes it possible to carry out an additional independent verification of the Pomeranchuk theorem. Owing to the absence of models for the description of the amplitudes in the low-energy and asymptotic energy regions, the d.r. for $K^\pm d$ scattering was not used before.

The amplitudes $f_\pm = D_\pm + iA_\pm$ for $K^\pm d$ scattering forward satisfy the d.r. [4]

$$D_\pm(\omega) = I_\pm(\omega) + \frac{k^2}{4\pi^2}\int_{\omega_0}^{\infty}\frac{d\omega'}{k'}\left[\frac{\sigma_+(\omega')}{\omega' \mp \omega} + \frac{\sigma_-(\omega')}{\omega' \pm \omega}\right], \tag{1}$$

where all the quantities are expressed in the laboratory system. The terms $I_\pm$ contain unknown subtraction constants and dispersion integrals up to an energy ω_0 = 0.79 GeV, above which the total cross sections $\sigma_\pm$ are known from experiment [4, 5]. Using the available data up to 20 GeV for $K^\pm d$ and up to 55 GeV

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