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"SUPERHETERODYNE" AMPLIFICATION OF ULTRASOUND IN SEMICONDUCTORS

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We consider in this paper the effect of distributed "superheterodyne" amplification of sound by a drift current of electrons in a semiconductor in the presence of an intense sound wave playing the role of a heterodyne. It will be shown that when the signal (carrier frequency ω_s) interacts nonlinearly with the heterodyne (frequency ω_h), the information in it is converted along the length of the crystal into another frequency (intermediate, ω_i), and vice versa. As a result, if the growth increment possessed by the intermediate frequency is sufficiently large, it is transferred to the signal frequency.

As is well known, [1 - 3], the synchronism condition causes separation of a series of waves that interact most strongly via the pump (the ω_h) wave. These, for example, may be the waves with frequencies ω_s, $\omega_i = \omega_h - \omega_s$, $\omega_p' = \omega_h + \omega_s$, and other combination frequencies of the type $n\omega_h \pm \omega_s$ (n is the integer). It is important that this results in a system of coupled monochromatic waves. Each such monochromatic wave is not a natural wave of the crystal. A natural wave represents, generally speaking, a mixture of monochromatic waves of all the interacting frequencies and is characterized by a single growth increment and by a distinct matching of the phase velocities of the individual monochromatic components. In the general case this increment describes two effects, namely parametric amplification [1 - 3] and the usual linear amplification [4]. The natural wave, which goes over in the absnece of the pump ω_h into a wave with frequency ω_i, will be called for brevity "natural wave of type ω_i" (and similarly for waves of the type ω_s, ω_i', etc.).

Assume that elastic oscillations of frequency ω_s are excited at the entrance into the crystal. Then, owing to the parametric interaction with the pump ω_h, a superposition of different natural waves will be excited in the crystal. Let us assume further that the growth increment of one such wave (say ω_i) is much larger than of the others. This condition is easiest to realize far from parametric resonance ($\omega_s >> {}_i$ or $\omega_s << \omega_i$). It may then turn out that the admixture of the frequency ω_s in the natural wave of type ω_i will have a much larger amplitude at the exit from the crystal than the oscillations of frequency ω_s in the other natural modes. In this case amplification of sound of frequency ω_s will be determined by the maximum growth increment α_i of the wave of type ω_i.

The described amplification mechanism may have nothing in common with ordinary parametric amplification. Indeed, let us consider a case in which the pumping is intense enough to ensure effective coupling between the oscillations of frequency ω_s and ω_i, but is at the same time weak enough to be able to neglect the effect of parametric amplification - the influence of the pump on the growth increment ω_i. That such a situation is indeed possible can be demonstrated by calculation and will be shown later. Then the amplification of the wave occurs almost exclusively as a result of supersonic electron drift [4]. The action of the wave ω_h, on the other hand, reduces to a transfer of the large linear gain, which is inherent in the frequency ω_i, to the signal frequency ω_s, The wave ω_h acts therefore in this case exactly like a heterodyne wave.

The theory of such an amplification can be constructed in analogy with the theory of parametric amplification of sound by sound in semiconductors (cf., e.g., [3]). Let us consider, just as in [3], for simplicity the interaction of three waves having the same polarization, with frequencies ω_s, ω_h, and $\omega_i = \omega_h - \omega_s$ and wave vectors $\vec{q}_s$, $\vec{q}_h$, and $\vec{q}_i = \vec{q}_h - \vec{q}_s$. We assume all three waves to be sufficiently weak in the sense of applying to them the method of amplitude iteration. We assume that the heterodyne amplitude is much larger than the amplitudes of the signal and of the intermediate frequencies, and we neglect on this basis the reaction of the latter on the heterodyne. In addition, we consider only the case when $\omega_s \sim \omega_h \gg \omega_i$ and the maximum linear gain corresponds to to the frequency ω_i. This allows us to neglect the gain of the heterodyne, thereby simplifying the calculation. We then obtain a system of two coupled linear differential equations of first order for the complex amplitudes of the signal $u_s(x)$ and of the intermediate frequency $u_i(x)$ as functions of the spatial coordinate x. We seek for these equations a solution satisfying the boundary condition $u_s(0) = U_0$ and $u_i(0) = 0$. We then calculate the gain in dB/cm

$$\gamma = \frac{20}{4} \lg \left| \frac{u_c(L)}{u_c(0)} \right|,$$

where L is the length of the crystal.

As a result we obtain

$$\gamma = 8{,}6 \left[\alpha_i + \frac{1}{L} \ln \left| \frac{|g|^2 K_i K_s^* - e^{-(\alpha_i - \alpha_s) L}}{1 - |g|^2 K_i K_s^*} \right| \right], \tag{1}$$

where α_s and α_i are the growth increments of the natural waves of type ω_s and ω_i, respectively, $g = \delta N/N_0$, δN is the complex amplitude of the oscillations of the electron density in the heterodyne wave, and N_0 is the equilibrium electron density. The quantities K_i and K_s^* are distribution coefficients, namely $igK_i = u_s/u_i^*$ in the natural wave of type ω_i and $-ig^*K_s = u_i^*/u_s$ is the natural wave of type ω_s. If the intensity of the heterodyne wave is $\sim|g|^2 \to 0$, then we obtain from (1) $\gamma = 8.6\alpha_s$ dB/cm. In other words, the signal is amplified in accordance with the linear theory [4]. When

$$|g|^2 \sim |g|^2_{cr} = \frac{1}{|K_i K_s^*|} e^{-(\alpha_i - \alpha_s) L} \tag{2}$$

the superheterodyne amplification mode sets in (it is assumed that $\alpha_i >> \alpha_s$ and $\alpha_i L >> 1$). It is seen from (2) that at large values of $\alpha_i L$ the parameter $|g|^2_{cr}$ is exponentially small. Since the parametric increment of the gain far from resonance ($\omega_s >> \omega_i$) is proportional to $|g|^2$, it can actually be small compared with α_i. Then α_i is in practice the growth increment [4]. Let us take, by way of an example

$$q_i r_D = 1, \quad \omega_i \tau_M\left(\frac{v_0}{v_s}\right) - 1 \sim 0{,}1 ,$$

$\alpha_i/\alpha_s \sim 0.3$ and $q_i L \sim 10^4$, where r_D is the screening radius, τ_M the Maxwellian relaxation time, v_0 the drift velocity, and v_s the speed of sound ($v_0 > v_s$). With these parameters we get from (2) $|g|_{cr} \sim 10^{-2}$. When $|g| > |g|_{cr}$ we get from (1) $\gamma = 8.6\alpha_i$ dB/cm, i.e., the maximum growth increment is indeed transferred to the dignal frequency. Such superheterodyne amplification was apparently first observed experimentally in [5][1], although the interpretation proposed by the authors of [5] requires further refinement. In addition, the case considered by us is apparently more interesting from the practical point of view, since in our case a large gain is transferred at high frequency.

We note that the principle of the distributed superheterodyne amplification is applicable also to other types of waves (for example, electromagnetic) propagating in a nonlinear medium capable of selective amplification of certain frequencies.

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