on. If the noise level due to the buildup of oscillations of other types [8] is insufficient, it is necessary to produce the initial perturbation purposefully, for example, by an initial perturbation of the concentration (current) of the beam, $\delta n_1 = A \exp(i\ell\theta)$.

It is also easy to obtain the buildup conditions for cases when the system responds to a perturbation not of the potential (field) but of some other quantity. For example, if $S = \Omega(\omega)\delta n_1$, the dispersion equation is obtained from the equation for the "free" beam by replacing $\omega - k_z u$ in the beam terms by $\omega - k_z u - i\Omega(\omega)$.

I am grateful to M.V. Nezlin for discussions.

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PHOTON EMISSION BY FAST CHARGED PARTICLES IN A SINGLE CRYSTAL

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Submitted 2 April 1970
ZhETF Pis. Red. 11, No. 10, 503 - 505 (20 May 1970)

1. It is known that a charged particle can emit a photon only by interacting with a third body (an atom of matter). For electrons, the most important process is the one in which the quantum is emitted directly by a fast particle (bremsstrahlung). For heavy particles, an important role is played by a mechanism in which a quantum is emitted by an electron of matter as a result of its scattering the particle's own field [1]. For example, for hard quanta, the cause of the radiation can be Compton scattering by the bound electrons. As shown in [2], in an amorphous medium this mechanism becomes predominant, even for electrons, at energies $E \geq 10^5$ eV and frequencies $\omega \leq 10^7$, owing to the suppression of the bremsstrahlung by the polarization of the medium [3].

We consider below a mechanism analogous to that of [2] for radiation in a single crystal. An essential feature of a single crystal is the strong enhancement of Compton scattering in the region of small angles, for which the wavelength transmitted in the longitudinal direction (a quantity inverse to the longitudinal momentum transfer) exceeds the lattice constant a. Indeed, the Hamiltonians of the interaction between the quantum and the atoms is the sum of the Hamiltonians of the interaction with each atom. Then, neglecting the interaction of the atoms with one another, we can write the matrix element for the scattering of a quantum by the entire crystal in the form of the sum of the matrix elements for the scattering by each atom:

$$M = M_0 \sum_{a=1}^{N} \exp i(\mathbf{k}-\mathbf{k}_0)\mathbf{R}_a .$$

Therefore the cross section for the scattering of the quantum by a system of N atoms is the product of the cross section for scattering by a single atom and the crystal factor

$$d\sigma_N = d\sigma_1 \left| \sum_a \exp[i(\mathbf{k}-\mathbf{k}_0)\mathbf{R}_a] \right|^2 .$$

For scattering angles $\theta \ll m/\omega$ the crystal factor takes the form

$$\left| \sum_a e^{i(\mathbf{k}-\mathbf{k}_0)\mathbf{R}_a} \right|^2 = N_y N_z \left(\frac{2\pi}{a}\right)^2 \sum_{m,n} \delta(k_y - k_{0y} - \frac{2\pi}{a} n)\, \delta(k_z - k_{0z} - \frac{2\pi}{a} m) \times$$
$$\times \left\{ \sin^2\left[N_x \frac{(k_x - k_{0x})a}{2} \right] \Big/ \sin^2\left[\frac{(k_x - k_{0x})a}{2} \right] \right\}$$

(we are considering a single crystal with cubic lattice and with dimensions $N_x a$, $N_y a$, and $N_z a$, and with N_y, $N_z \to \infty$). When $N_x a(k_x - k_{x0}) \ll 1$ (thin layer the crystal factor is proportional to $N_x^2 N_y N_z$, i.e., the amplitudes are additive in the longitudinal directions, and not the scattering probabilities. Consequently, the cross section for small-angle scattering of the quantum in the single crystal increase in comparison with scattering in an amorphous medium by a factor N_x. The region of effective scattering angles of the quantum in the single crystal shifts in the direction of smaller angles.

2. The features of the scattering of a quantum in a single crystal may come into play in processes for which the small-angle scattering region is important, particularly for emission of a quantum when a fast particle moves in a single crystal, if the radiation is caused by Compton scattering of the particle's own field (in other words, emission of recoil electrons).

The emission cross section can be represented in the form

$$d\sigma(\omega) = \int d^3q\, d\sigma_0(\omega, q) \left| \sum_a \exp(i\, \mathbf{q}\mathbf{R}_a) \right|^2 ,$$

where $d\sigma(\omega, q)$ is the cross section for the emission of a quantum by the recoil electron in the interaction of the particle with one atom, and q is the momentum transferred to the crystal by the quantum upon scattering. Introducing the direction cosines α, β, and γ of the particle motion in the single crystal, we obtain

$$d\sigma^{cr}(\omega_2) = 2\pi r_0^2 \mu \frac{N_\perp Z^2 a}{a^2} d\omega_2 \int \frac{d\omega_1}{\omega_1^3} \left[\frac{\omega_1}{\omega_2} + \frac{\omega_2}{\omega_1} + \left(\frac{\mu}{\omega_2} - \frac{\mu}{\omega_1}\right)^2 - \right.$$
$$\left. - 2\left(\frac{\mu}{\omega_2} - \frac{\mu}{\omega_1}\right)\right] \sin^2 N_\parallel \frac{a}{2a}\left[\frac{\mu}{E}(\omega_1 - \omega_2) - \frac{2\pi}{a}(m\beta + \ell\gamma)\right] : \sin^2 \frac{a}{2a}\left[\frac{\mu}{E}(\omega_1 - \omega_2) - \frac{2\pi}{a}(m\beta + \ell\gamma)\right] \times$$
$$\times \exp\left\{ -\left[\left(\frac{2\pi}{a}\right)^2 (m^2 + \ell^2) + \frac{\mu^2}{E^2}(\omega_1 - \omega_2)^2 \overline{u^2}\right]\right\} \int \frac{dk_2 dk_3 (k_2^2 + k_3^2)\, \delta^2(k_\perp - \frac{2\pi}{a} n_\perp)}{\left(k_2^2 + k_3^2 + \frac{\omega_1^2 \mu^2}{E^2}\right)^2} ,$$

where we have also taken into account the influence of the temperature oscillations of the lattice atoms. If $\sigma\mu(\omega_1 - \omega_2)/E \ll 1$ (the longitudinal transmitted wavelength exceeds the lattice constant), then the emission cross section is given by

$$d\sigma^{cr}(\omega_2) = d\sigma^{am}(\omega_2) N_{\parallel} \left\{ 1 - \left[1 + 3\left(1 - \frac{E}{\mu\omega_2 N_{\parallel} a}\right)^2 \right] \Big/ 4\left(1 - \frac{E}{\mu\omega_2 N_{\parallel} a}\right)^3 \right\},$$

where $d\sigma^{am}(\omega_2)$ is the cross section for emission in an amorphous medium. For a thin single crystal ($N_{||} < E/\mu\omega a$), the quantum-emission cross section increases appreciably compared with the amorphous medium. The ratio of the cross sections in the crystal and in the amorphous medium is

$$d\sigma^{cr}(\omega_2) / d\sigma^{am}(\omega_2) \sim N_{\parallel} \gg 1.$$

The indicated mechanism of radiation from a heavy relativistic particle in a single crystal predominates in the frequency region

$$\omega < m(\mu/m)^2 .$$

The authors are grateful to V.M. Galitskii for a useful discussion of the work.

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PRODUCTION OF NEUTRAL VECTOR MESONS IN THE PROCESSES $e^- + e^+ \to \rho^0(\omega^0, \phi^0) + \gamma$

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ZhETF Pis. Red. 11, No. 10, 505 - 508 (20 May 1970)

At the present time, experiments on colliding electron-positron beams, performed in Novosibirsk and in Orsay [1, 2], offer evidence that the cross sections of the processes $e^-e^+ \to \pi^-\pi^+$, $e^-e^+ \to \pi^-\pi^+\pi^0$, and $e^-e^+ \to K_L^0K_S^0(\pi^-\pi^+\pi^0)$ have a resonant character at a summary initial-particle energy near the masses of the ρ^0, ω^0, and ϕ^0 messons, respectively. With increasing energy, the cross sections of these processes go outside the resonant region and drop rapidly, in view of the strong decrease of the electromatnetic form factors. The radiative corrections to the resonant annihilation of the electron and positron at an initial-particle energy close to the mass of the vector resonance were considered in [3]. It was shown there that the radiative correction contains, besides the electromagnetic constant α, also a large factor $\ln s/m_e^2$, where $s = 4E^2$, E is the energy of the initial particle in the c.m.s. of the electron and positron, and m_e is the electron mass. In addition, the emission of a photon causes the production of the pseudoscalar mesons to proceed via a vector resonance even if the total energy of the initial particles does not coincide with the mass of the vector mesons.

2. The foregoing considerations remain in force also with increasing energy E, when a hard photon is emitted. We are interested in this paper in