WEAK TURBULENCE AND ANOMALOUS DIFFUSION

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1. According to the premises established by now, the development of unstable oscillations in a plasma can lead to establishment of a stationary turbulent state. The degree of development of the turbulence is characterized by a correlatinn time t_c, which is assumed equal to the reciprocal increment, $t_c \simeq \gamma^{-1}$. The turbulence is assumed to be weak if $\omega t_c >> 1$, and strong if $\omega t_c \simeq 1$, where ω is the oscillation frequency. There is no doubt that strong turbulence should actively contribute to the establishment of thermodynamic equilibrium. It is assumed [1] that the analogous action of weak turbulence is weakened by a factor ωt_c. These considerations are usually employed to estimate turbulent transport coefficients, such as the coefficient of diffusion due tp drift oscillations. An analysis of the kinetic equation for the waves confirms the possibility of such an approach [2]. We note that the latter has never been solved accurately because of its complexity.

In our opinion, however, a correct determination of the transport coefficients is impossible without knowledge of the spectrum of the unstable oscillations. Using as an example the diffusion due to low-frequency oscillations of the drift type, it is shown that the customarily employed values of the diffusion coefficient are apparently much too high. To obtain this result, let us consider the change of the dispersion of the coordinates of the plasma particles under the influence of random electric fields. An equivalent approach is used in the theory of turbulence of an ordinary liquid, and some of the formulas of this theory are the analogs of the formulas presented below, see, for example, [3].

2. Let us consider plasma diffusion in a magnetic field, produced by random electric fields with frequency ω much lower than the cyclotron frequency ω_c. The equation of motion in the coordinate system in which the average velocity is zero is of the form

$$\dot{\mathbf{r}} = \frac{c}{H^2}[\mathbf{H} \times \mathbf{E}(t)] \ . \quad (1)$$

Taking into account the connection between the diffusion coefficient D and the change of the variance of the coordinates of the individual particles, $D = 2(d/dt)\langle \vec{r}^2 \rangle$, we obtain

$$D = 4(c^2/H^2) \int_{-\infty}^{0} k(t)\, dt. \quad (2)$$

Here $K(t) = \langle \vec{E}(0)\vec{E}(t) \rangle$ is the correlation function and the brackets denote averaging over the ensemble. We consider weak turbulence, and therefore $K(t) = F(t/t_c)\cos \omega t$, where $\omega t >> 1$. We expand $F(t/t_c)$ in powers of

$$F(t/t_c) = \sum_{n=0}^{\infty} (a_n/n!)(t/t_c)^n,$$

and then we obtain from (2) (see [4]):

$$D = 4(c^2/H^2)(1/\omega) \sum_{n=0}^{\infty} [a_n/(\omega t_c)^n] \cos\left(\frac{\pi}{2}(n+1)\right) . \quad (3)$$

The even terms with $n = 2k$ vanish in this case. The sum over the modes of the oscillations has been omitted for brevity. If we assume that α_0 and α_1 are of the same order of magnitude and recognize that $\alpha_0 = k(0) = \langle \vec{E}^2 \rangle$, then we can readily obtain with the aid of (3) the customary expression for the diffusion coefficient $D = \gamma^2/\omega k^2$ (see, e.g., [1]). However, for a real stationary random process $\vec{E}(t)$, corresponding to an even correlation function $K(t) = K(-t)$, the condition $\alpha_1 \neq 0$ would mean that the quantity $\partial\vec{E}/\partial t$ does not exist (see, e.g., [5]). Yet this quantity has a physical meaning, since it enters in Maxwell's equation. At the same time there is little likelihood that all the coefficients α_{2k+1} vanish, since the turbulence would then be "quasirandom," since the prior history of the process $\vec{E}(t)$ would uniquely determine its further variation (see [5]). Thus, apparently some of the coefficients α_{2k+1} ($k \geq 1$) do not vanish, and consequently

$$D \approx \frac{4}{\omega} \frac{c^2}{H^2} \frac{\alpha_{2k'+1}}{(\omega t_c)^{2k'+1}},$$

i.e., smaller than the customarily employed value by a factor $(\omega t_c)^{2k'}$. Here $k' = k_{min}$, at which $\alpha_{2k+1} \neq 1$.[1)]

If we assume nevertheless that the turbulence is "quasirandom," $F(t/t_c) = f(t^2/t_c^2)$, then we obtain exponentially small values of D^c at $\omega t_c \gg 1$. Thus, for example, at $F(t/t_c) = \alpha_0/(1 + t^2/t_c^2)$ we have

$$D = 2\pi \frac{c^2}{H^2} \alpha_0 t_c e^{-\omega t_c}. \qquad (4)$$

In final analysis, turbulent diffusion is due to resonant interaction between oscillations and the plasma particles. Indeed, with the aid of the relation

$$S(\omega') = (1/2\pi)\int_{-\infty}^{\infty} dt \cos\omega t\, K(t),$$

where $S(\omega')$ is the spectral density (intensity) of the oscillations, we can transform (2) into

$$D = 4\pi \frac{c^2}{H^2} S(0). \qquad (5)$$

We recall that we are using a coordinate system in which the particles are on the average at rest, and therefore oscillations with $\omega = 0$ resonate with the particles. The resonant interaction is appreciable when the spectral lines are strongly broadened.

In conclusion, we state without proof some simple results. 1) Allowance for the inertia in (1) does not change the value of D up to second order in ω/ω_c inclusive. 2) A procedure analogous to that described above makes it possible to obtain the diffusion coefficient in velocity space; this coefficient determines the intensity of the heating of the plasma by the oscillations.

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1) If all $\alpha_{2k+1} = 0$, then the spectral density $S(\omega')$ decreases exponentially $|\omega - \omega'| \to \infty$: if $\alpha_{2k'+1} \neq 0$, then $S(\omega') \sim (\omega - \omega')^{-2(k'+1)}$ when $|\omega - \omega'| \to \infty$ and the random process is of the Markov type [5], i.e., it certainly is irreversible.

3) If the weak turbulence is "quasirandom," then there is practically neither diffusion nor heating. 4) Our analysis does not apply to the results of quasi-linear theory and the theory of strong turbulence.

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FEATURES OF ELECTRON DRAG OF DISLOCATIONS IN SUPERCONDUCTORS

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Several experimental groups have recently observed an influence of the superconducting transition on the kinetics of plastic deformation of metals [1 - 3][1]. Since the properties of the crystal lattice remain practically unchanged in a superconducting transition, it is natural to ascribe the observed effect to the influence of conduction electrons. The possible mechanism of such an influence is as follows. As is well known, the plastic properties of crystal are determined in final analysis by the mobilities of the individual dislocations. Kravchenko has shown earlier [4] that a dislocation moving in a normal metal experiences a drag force caused by its interaction with the conduction electrons. This force has an appreciable magnitude and its variation, expected in the case of the superconducting transition, can greatly influence the balance of forces that determine the dislocation mobility. In this communication we present results of an investigation of the force of electron drag by dislocations in superconductors. It will be shown below that when the metal goes over from the normal to the superconducting state the character of the variation of this force is not as simple as assumed earlier [4, 5], but reveals a number of interesting features.

A dislocation moving with constant velocity $\vec{V}$ produces in a crystal an alternating field of elastic deformations $u_{in}(\vec{r} - \vec{V}t)$, which exerts on the conduction electron a force determined by the deformation potential $\lambda_{in}u_{in}(\vec{r} - \vec{V}t)$ (the components of the tensor λ_{in} are of the order of the Fermi energy ε_F [6]). Because of this potential, a moving dislocation produces transitions in the electron system and loses its energy to the perturbation of this system. The dislocation retardation force due to such losses is equal in absolute magnitude to the energy absorbed by the electrons when the dislocation moves on a unit path.

It can be shown that the Hamiltonian of the interaction between the electrons and the moving dislocation has in the second-quantization representation the form

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[1]The cited articles are not the only publications in which the influence of the superconducting transition on plastic properties of metals is noted. We have cited only the investigations in which, in our opinion, the effect is most clearly pronounced.