If $T \neq 0$, but $T << \Delta$ and $V < V_{cr}$, we have

$$F(V) \approx \begin{cases} \dfrac{n\epsilon_F b^2 q_m}{\pi v_F} e^{-\Delta/T} V, & \hbar q_m V << T << \Delta \\ \dfrac{n\epsilon_F b^2}{\pi\hbar v_F} \sqrt{T}\, e^{-\Delta/T} \sqrt{\hbar q_m V}, & 2\Delta > \hbar q_m V >> T \end{cases} \qquad (9)$$

Attention must be called to the fact that F(V) becomes nonlinear at sufficiently high velocities.

Formulas (6) - (9) show that the force of the electron drag by dislocations in a superconductor has a complicated dependence on the velocity and on the temperature.

We take the opportunity to thank V.P. Galaiko, V.L. Pokrovskii, and V.Ya. Kravchenko for useful discussions of the results of the work, and also V.B. Fiks, with whom problems close to those touched upon in the present communication were discussed.

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CONTRIBUTION TO THE THEORY OF THE MIXED STATE OF SUPERCONDUCTORS OF THE SECOND KIND

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Submitted 18 March 1970; resubmitted 28 April 1970
ZhETF Pis. Red. 11, No. 11, 554 - 556 (5 June 1970)

A method is proposed for describing a superconductor in the case when the Cooper pairs can form several condensates, i.e., they can accumulate in several different states. This problem is meaningful in the presence of degeneracy of the ground state of the Cooper pair, as is the case in a superconductor of the second kind in a magnetic field.

The starting point is the Ginzburg-Landau theory [1]. In this theory, the free energy is

$$F = F_0 + \int d^3r \left\{ \frac{1}{2m} |(\nabla - i\frac{e}{c} A)\psi|^2 + a|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \frac{H^2}{8\pi} \right\}, \tag{1}$$

where F_0 is the free energy of the normal metal in the absence of a magnetic field, H is the magnetic field, A is the vector potential, and ψ is the wave function of the pair as a whole (e is double the electron charge).

The Ginzburg-Landau theory is not suitable for a situation in which several states can become filled simultaneously, say incoherently. We assign to the Bose-particle system (Cooper-pair system) the Hamiltonian

$$\hat{\mathcal{H}} = \int d^3r \left\{ -\frac{1}{2m} \hat{\psi}^+ (\nabla - i\frac{e}{c} A)^2 \hat{\psi} + a\hat{\psi}^+\hat{\psi} + \frac{\beta}{2} \hat{\psi}^+\hat{\psi}^+\hat{\psi}\hat{\psi} \right\}, \tag{2}$$

which is obtained from (1) by making the substitution $\psi \to \hat{\psi}$, where $\hat{\psi}$ is the Bose-type operator (it is defined below).

The possibility of such an assignment is our main premise, which is supported by the following argument. As shown in [2], the phase transition in a superconductor is equivalent to a transition in a system of interacting Bose particles. The role of the Bose particles is played by the Cooper pairs. It can be verified that the Hamiltonian of such a system coincides with (2).

Let us consider a superconductor of the second kind in a magnetic field. We assume that the magnetic field inside the superconductor is constant, as is confirmed by the result. Then the operator $\hat{\psi}$ can be written in the form

$$\hat{\psi} = \sum_k \hat{a}_k \psi_k , \tag{3}$$

where ψ_k is an orthonormal set of wave functions of the ground state of the particle in a magnetic field, i.e., ψ_k are solutions of the equation

$$-\frac{1}{2m}(\nabla - i\frac{e}{c} A)^2 \psi_k = \frac{H}{H_{c2}} |a| \psi_k \tag{4}$$

(the momentum of the particle along the field is equal to zero). The operators $\hat{a}_k$ and $\hat{a}_k^+$ are the ordinary Bose operators of annihilation and creation of a particle in a state ψ_k with commutation relations

$$[\hat{a}_k, \hat{a}_{k'}^+] = \delta_{kk'}, \quad [\hat{a}_k, \hat{a}_{k'}] = 0. \tag{5}$$

In expansion (3) we took into account only the ground-state levels, since the excited states play a role in a very narrow region near the transition point (see, e.g., [2]).

It turns out that the ground state of the operator $\hat{\mathcal{H}}$ (2) corresponds to the wave function

$$\phi = \hat{\psi}^+(r_1)\, \hat{\psi}^+(r_2) \ldots \hat{\psi}^+(r_N) |0\rangle , \tag{6}$$

where the points $r_1, r_2, \ldots, r_N$ are uniformly distributed over the cross section S of the superconductor ($\hat{\psi}$ does not depend on the coordinate along H), and N is the number of particles. The number of such points is proportional to the volume V, and the two-dimensional density is proportional to V/S, so that the average distance between points tends to zero in the limit of a large volume.

In the wave function (6) there are no preferred points, so that the density is constant, and there are no preferred directions perpendicular to H, and therefore the currents are equal to zero, i.e., H = const inside the superconductor.

The mean value of the operation (2) over the state (6) is

$$<\hat{\mathcal{H}}> = a\left(1 - \frac{H}{H_{c2}}\right)N + \frac{\beta}{2}\frac{N^2}{V} \quad . \tag{7}$$

The validity of this expression can be verified by writing the operator (2) and the wave function (6) in the coordinate representation, and taking into account the fact that for a Bose system it is possible to use in the variational principle a function that is not symmetrical with respect to the coordinates of the particles (such a function is chosen to be one of the terms in the expression for ϕ in the coordinate representation). The proof of (7) and a more detailed discussion of the questions considered here will be published elsewhere.

The free energy of the system is

$$F = F_0 + <\hat{\mathcal{H}}> + \int \frac{H^2}{8\pi} d^3r \, . \tag{8}$$

The minimum value of this quantity at given H is

$$\frac{F - F_0}{V} = - \frac{H_c^2}{8\pi}\left(1 - \frac{H}{H_{c2}}\right)^2 + \frac{H^2}{8\pi} \, , \tag{9}$$

where $H_c^2/8\pi = \alpha^2/2\beta$.

Comparison with the results of Abrikosov for a vortex structure [3] shows that the homogeneous mixed state (6) is more favorable. Thus, for example, near H_{c2}, we have for a vortex structure [3]

$$\left(\frac{F - F_0}{V}\right)_{\text{vort}} = \frac{-H_c^2}{8\pi}\left(1 - \frac{H}{H_{c2}}\right)^2 \frac{2\kappa^2}{1 + (2\kappa^2 - 1)\beta_0} + \frac{H^2}{8\pi} \, , \tag{10}$$

where κ is the parameter of the Ginzburg-Landau theory ($\kappa > 1/\sqrt{2}$ for superconductors of the second kind), $\beta_0 > 1$ for any lattice ($\beta_0 = 1.16$ for a triangular lattice), and H is in this case the average magnetic field inside the superconductor.

The single-particle density matrix of the state (6) is

$$< \hat{\psi}^+(r') \, \hat{\psi}(r) > \sim \sum_k \psi_k^*(r') \, \psi_k(r) \, . \tag{11}$$

This quantity decreases exponentially at distances on the order of $(c/eH)^{1/2}$. It can therefore be stated that in the state (6) there is no long-range order in the customarily understood sense.

A homogeneous mixed state takes place in the interval $H_{c1} < H_0 > H_{c2}$ (for a bulky long cylinder along the field; H_0 is the external field), with

$H'_{c1} = H_c/\sqrt{2}\kappa$ and smaller than the lower limit for the vortical state H_{c1}. A second-order phase transition takes place at the points H'_{c1} and H_{c2}.

This raises the question of how the magnetic field penetrates into the bulky sample when the external field changes. This proceeds, apparently, via formation of a nucleus near the surface, with a transverse dimension on the order of the characteristic dimension of (11), i.e., $(c/eH)^{1/2}$. When $H_0 \to H'_{c1}$ the dimension of the nucleus is large, since $H \to 0$. A homogeneous mixed state can therefore hardly be realized in a bulky cylinder, except in the vicinity of H_{c2}.

The question of penetration of the field does not arise in the case of a thin plate transverse to the field (the plate dimension along the field is small compared with the remaining dimensions, since the external field is not screened, $H = H_0$. A homogeneous state should therefore be realized here, rather than a vortical one (in the case of a plate, the energy of the vortical plate (10) increases also as a result, say, of the field inhomogeneity near the plate surface). This is the most suitable object for an experimental verification of the theory.

I am grateful to A.I. Larkin and V.L. Pokrovskii, and also to I.E. Dzyaloshinskii, A.P. Kazantsev, and G.M. Eliashberg, for useful discussions and interesting remarks.

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PROTON-NEUTRON CORRELATIONS IN MEDIUM AND HEAVY NUCLEI

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Submitted 4 May 1970
ZhETF Pis. Red. 11, No. 11, 557 - 559 (5 June 1970)

It is well known (cf., e.g., [1]) that it follows from a comparison of data on hadron and lepton scattering that the radius R_Z of the charge distribution in the nucleus is smaller than the radius R_A of mass distribution. As noted recently in [2], the corresponding experimental data fit the formula

$$R_z = r_0 (2z)^{1/3}; \qquad R_A = r_0 A^{1/3}, \tag{1}$$

where $r_0 = (5/3)^{1/2}$F. According to (1), the densities and hence the chemical potentials of the proton and neutron distributions are equal (but not the radii of these distributions). Apparently, the fact whether relation (1) is satisfied or not should strongly influence certain conclusions of the superconducting model of the nucleus.

The point is that in the analysis of medium and heavy nuclei it is customary to start with the model of two superconducting liquids (neutron and proton). This is connected with the fact that if the difference between the radii of the neutron and proton distribution radii is not taken into account, then the difference of the chemical potentials of the neutrons and protons turns out to be $\Delta\zeta \sim 5 - 10$ MeV. This is larger than the expected energy gap in the spectrum, $2\Delta \sim 2 - 3$ MeV, and is usually the justification for not taking into