NEW TYPE OF QUANTUM ELECTROMAGNETIC WAVES IN A STRONG MAGNETIC FIELD

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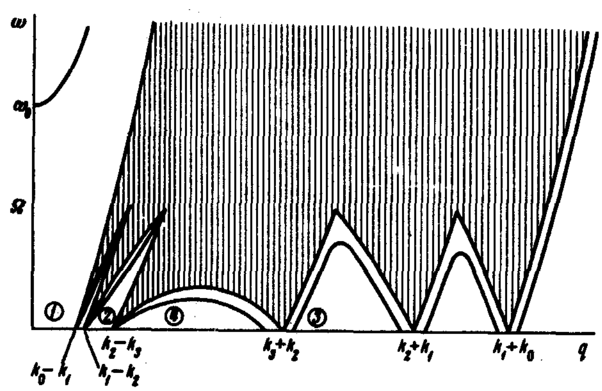
It is shown in a number of theoretical papers that in a degenerate electron gas there can exist longitudinal and transverse electromagnetic oscillations due to quantization of the orbital motion of electrons in a strong magnetic field [1 - 4]. These oscillations, called quantum waves, are connected with definite resonant transitions between the Landau levels n. Each single-particle resonance with specified Δn corresponds to one Bose branch of the collective excitations in the system of interacting electrons. If the quantization conditions $\hbar\Omega \gg T$ and $\Omega \gg \nu$ are not satisfied, then the quantum waves attenuate strongly ($\hbar$ - Planck's constant, Ω - cyclotron frequency, T - temperature in energy units, ν - collision frequency). In this paper we find the dispersion laws of low-frequency right-polarized electromagnetic waves propatating along a magnetic field.

The dispersion curves of the quantum electromagnetic excitations lie in those sections of the (ω, q) plane where there is no Landau damping. Then all the electron transitions with specified Δn correspond to definite regions of Landau damping. By virtue of the Kramers-Kronig dispersion relations, the imaginary part of the conductivity is singular at the boundaries of the damping regions when $T \to 0$ and $\nu \to 0$. It follows from the dispersion relations that near the singularity of Im σ_{ik} there appears, as a rule, a solution corresponding to a definite type of electromagnetic oscillations.

The Landau damping regions can be obtained from an analysis of the conservation laws and the Pauli principle (see [5, 6]). It can be shown that for right-polarized waves propagating along the magnetic field, when $\Delta n = -1$, the Landau-damping regions are determined by the inequalities (see the figure):

$$\begin{aligned} \omega &\geq -\Omega - \frac{\hbar q^2}{2m} + v_n q, \\ \omega &\leq -\Omega + \frac{\hbar q^2}{2m} + v_n q, \\ \omega &\geq -\Omega + \frac{\hbar q^2}{2m} - v_n q. \end{aligned} \qquad (1)$$



Schematic representation of the dispersion curves and of the Landau damping region for right-polarized excitations.

As seen from the figure, when $\omega < \Omega$, there exist many sections where there is no damping. Different types of these sections are marked by the figures 1, 2, 3, and 4. If the electrons in the crystal occupy N Landau levels, then there exist N - 2 damping-free regions of the second type, and N - 2 regions of the third type. (The figure shows the case when four Landau levels are occupied.) The height of the section numbered 4 is determined by the degree of occupation of the highest Landau level.

Near the boundaries of the damping-free sections of the second, third, and fourth type, there should exist solutions of the dispersion equation for right-polarized waves. Let us demonstrate this analytically. A general expression for the conductivity tensor σ_{ik}, under the Landau quantization conditions, was obtained in [7, 8]. Following [8], we can show that for a right-polarized wave and $\vec{q} \,||\, \vec{H}$ the dispersion equation is given by

$$-4\pi^2 \ell_H^4 n_0 q \left(1 - \frac{\omega^2 - c^2 q^2}{\omega_p^2}\right) = \sum_{n=0}^{N} \ln \left\{ \left| \frac{v_n q - \frac{\hbar q^2}{2m} - (\omega + \Omega)}{v_n q + \frac{\hbar q^2}{2m} + (\omega + \Omega)} \right|^{n+1} \times \left| \frac{v_n q - \frac{\hbar q^2}{2m} + (\omega + \Omega)}{v_n q + \frac{\hbar q^2}{2m} - (\omega + \Omega)} \right| \right\}, \qquad (2)$$

where ω_p is the plasma frequency, $\ell_H = (c\hbar/eH)^{1/2}$ is the magnetic length, $v_n = \hbar k_n/m$ is the maximum velocity of the electron at the level n, and n_0 is the concentration. Equation (2) is written for the condition T = 0 and $\nu = 0$.

Let us find the solutions of the dispersion equation in the sections of the second type. Separating in the right side of (2) the term leading to a singularity on the boundaries of the n-th section and denoting the contribution of the remaining terms by $b(\omega, q)$, we obtain, if $\omega \ll \Omega$,

$$\omega_n(q) = v_n \tilde{q} - (v_{n-1} - v_n)\tilde{q} \exp\{-f(q) + b\}, \qquad (3)$$

where

$$\tilde{q} = q - (k_n - k_{n+1}), \quad f(q) = \left(1 + \frac{c^2 q^2}{\omega_p^2}\right) 4\pi^2 \ell_H^4 n_0 q.$$

The quantity $b(q, \omega)$ can be estimated by replacing the summation in (2) with integration. For the parameters corresponding to a metal and to a strongly-degenerate semiconductor, $f(q) \gg b$ in the region under consideration, so that we can put b = 0 in (3). Analogously, separating the principal terms in the right side of (2), we obtain on the boundaries of the sections of the third type, if $\omega \ll \Omega$,

$$\omega_{1,2}(n, q) = \frac{1}{2}\{(v_n - v_{n+1})\tilde{q} \pm [(v_n + v_{n+1})^2 \tilde{q}^2 - 4a_n^2 \exp\{-f(q)\}]^{1/2}\}. \qquad (4)$$

Here $\tilde{q} = q - (k_n + k_{n+1})$, a_n changes from a quantity on the order of $\varepsilon_F/\hbar$ to Ω when n changes from zero to N - 2. Formulas (3) and (4) at n = N - 2 yield the solutions in region 4. The behavior of the solutions when $\omega \gtrless \Omega$ is shown in the figure.

Expanding the logarithms at small values of q, we obtain an equation

$$1 - \frac{\omega_p^2}{\omega^2 - \Omega^2} + \frac{\Omega}{\omega} \frac{\omega_p^2}{\omega^2 - \Omega^2} = \frac{c^2 q^2}{\omega^2}\left[1 - \frac{\beta_N \Omega^3}{(\omega + \Omega)^3} \frac{\omega_p^2 \ell_H^2}{c^2}\right], \qquad (5)$$

with

$$\beta_N = (4\pi^2 n_0)^{-1} \sum_{n=0}^{N} k_n^3,$$

which determines the weak renormalization of the "ordinary" wave [9]. For a calculation of $\sum_{n=0}^{N} k_n^2$ see [5].

To observe right-polarized quantum electromagnetic waves it is necessary, first, to satisfy the conditions for the existence of giant quantum oscillations of absorption [4]. Then the waves are weakly damped. In addition, for the existence of quantum waves it is necessary that the "resonant" logarithmic term at finite values of ν and T be comparable with the right-side of (2), meaning that in the relaxation-time approximation which we employ for the estimates it is necessary to satisfy the condition

$$\left| \ln \frac{\nu}{q v_n} \right| \gtrsim f(q) . \qquad (6)$$

This condition can apparently be satisfied in Bi and InSb, where $f(q) \approx 1$ at $H = 10^4 - 10^5$ Oe.

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CONCERNING THE EFFECT OF SCHWARZ AND HORA

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1. Recently Schwarz and Hora reported results of interesting experiments on the modulation of beam (E_0 = 50 keV), resulting from the passage of this beam through a thin crystal ($d = 10^{-5}$ cm) placed in the field of the light wave of a laser (λ = 4880 Å) [1]. Modulation of the electron beam at the optical frequency ω led to the appearance of glow of the same frequency when the beam struck a metallic screen. A system of light spots, corresponding to the directly transmitted beam, and also to the beams diffracted by the crystal, was observed on the screen. In this paper we explain these experiments and present results of a quantum theory of modulation of the electron beam. We confine ourselves only to the directly transmitted beam.

We note first that in the absence of a crystal the intersection of an electron beam with a laser beam cannot lead to modulation of the beam[1)], since absorption of one or several photons of laser radiation by the electron is forbidden by the conservation laws. The presence of the crystal makes this process possible, since the electron can acquire the required momentum from the crystal.

[1)]The reverse statement was made in [2].