which determines the weak renormalization of the "ordinary" wave [9]. For a calculation of $\sum_{n=0}^{N} k_n^2$ see [5].

To observe right-polarized quantum electromagnetic waves it is necessary, first, to satisfy the conditions for the existence of giant quantum oscillations of absorption [4]. Then the waves are weakly damped. In addition, for the existence of quantum waves it is necessary that the "resonant" logarithmic term at finite values of ν and T be comparable with the right-side of (2), meaning that in the relaxation-time approximation which we employ for the estimates it is necessary to satisfy the condition

$$\left| \ln \frac{\nu}{q v_n} \right| \gtrsim f(q) . \tag{6}$$

This condition can apparently be satisfied in Bi and InSb, where $f(q) \simeq 1$ at $H = 10^4 - 10^5$ Oe.

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CONCERNING THE EFFECT OF SCHWARZ AND HORA

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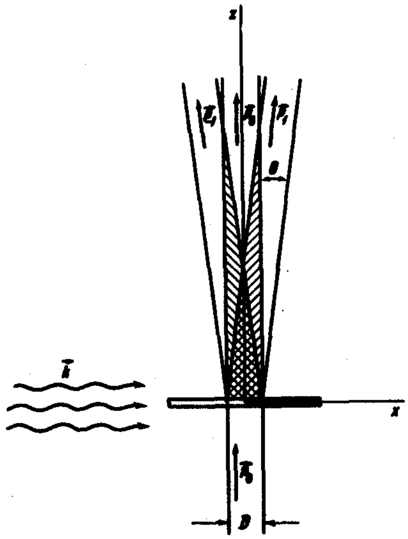
1. Recently Schwarz and Hora reported results of interesting experiments on the modulation of beam (E_0 = 50 keV), resulting from the passage of this beam through a thin crystal ($d = 10^{-5}$ cm) placed in the field of the light wave of a laser (λ = 4880 Å) [1]. Modulation of the electron beam at the optical frequency ω led to the appearance of glow of the same frequency when the beam struck a metallic screen. A system of light spots, corresponding to the directly transmitted beam, and also to the beams diffracted by the crystal, was observed on the screen. In this paper we explain these experiments and present results of a quantum theory of modulation of the electron beam. We confine ourselves only to the directly transmitted beam.

We note first that in the absence of a crystal the intersection of an electron beam with a laser beam cannot lead to modulation of the beam[1], since absorption of one or several photons of laser radiation by the electron is forbidden by the conservation laws. The presence of the crystal makes this process possible, since the electron can acquire the required momentum from the crystal.

[1]The reverse statement was made in [2].



2. In the classical description of the phenomenon, the beam modulation is the result of the fact that the normal component of the electric field of the wave experiences a jump on the surface of the crystal. Therefore the average force exerted on the electron by the field is not equal to zero; as a net result, the velocity of the electron changes, and this change depends on the phase of the field during the time of passage of the electron through the crystal. The decrease of the field intensity at the edges of the light beam occurs at distances much greater than the wavelength. Therefore the time of passage of the electron through the edge region is much larger than the period of the oscillations, and this region makes no contribution to the change of the electron velocity.

The resultant change of the electron energy, in accordance with the classical theory, is u sin ωt, where

$$u = \frac{\epsilon - 1}{\epsilon} \frac{2e\mathcal{E}_z v}{\omega} \sin \frac{\omega d}{2v} , \tag{1}$$

ε is the dielectric constant of the crystal, e is the charge of the electron, v is its velocity directed perpendicular to the surface of the crystal along the z axis, and $\mathcal{E}_z$ is the normal component of the electric field on the outer surface of the crystal.

If the quantity u defined by (1) is smaller than the quantum energy ℏω, then the classical description of the modulation is certainly not applicable. Under the experimental conditions of Schwarz and Hora, the ratio $u/\hbar\omega < 0.1$, so that a quantum-mechanical analysis is essential. We note that the classical description was used in [2].

3. According to quantum mechanics, an electron passing through a crystal illuminated by a laser does not have a definite energy or momentum. Its wave function is a superposition of states produced as a result of absorption or emission of n quanta hω (n = 0, ±1, ±2, ...). The modulation of the density and of the current of the electrons is due to the interference of these states. We present a solution of the Schrodinger equation for an electron incident along the z axis on a plate located in the plane z = 0. The laser beam is directed along the x axis and has an arbitrary linear polarization (see the figure). Ψ is the function of the electron passing through the crystal, and has outside the range of action of the laser-radiation field the form

$$\Psi(\mathbf{r}\ t) = \sum_{n=-\infty}^{\infty} J_n\left(-\frac{u}{\hbar\omega}\right)\exp\left(-i\frac{E_n t}{\hbar} + i\frac{\mathbf{p}_n \mathbf{r}}{\hbar}\right), \tag{2}$$

where J_n is a Bessel function of order n, $E_n = E_0 + nh\omega$, $p_n = (n\hbar k,\ 0,\ [p_0^2 + 2mn\hbar\omega - n^2\hbar^2k^2]^{1/2})$, and $E_0 = p_0^2/2m$ is the initial electron energy. The value of u is given by formula (1) as before. Modulation is produced only as a result of the wave's electric field component $\mathcal{E}_z$ normal to the surface of the crystal. This agrees with the data of [1]. The solution (2) is valid if the following conditions are satisfied: 1) the amplitude of the change of the electron velocity in the light wave is small compared with the initial velocity $e\mathcal{E}/p_0\omega \ll 1$, 2) the quantum energy is small compared with electron energy, $\hbar\omega/E_0 \ll 1$, 3) the wavelength of the electron is small compared with the characteristic distance, on the order of the atomic distance, in which the field changes at the surface of the crystal. These conditions are well satisfied as a rule; in particular, they are satisfied with a margin of several orders of magnitude in the experiment of [1].

4. In the experiment of Schwarz and Hora the parameter $u/\hbar\omega < 0.1$, so that only single-photon transitions were of importance ($n = 0, \pm 1$). We then obtain from formula (2) the current density per electron (at $z \ll (v/\omega)(E_0/\hbar\omega)^2 \sim 10^5$ cm)

$$j(r\ t) = ev\left[1 + 2\frac{u}{\hbar\omega}\sin\left(\frac{\omega}{v}\frac{\hbar\omega}{4E_0}z\right)\sin\omega\left(t - \frac{z}{v} - \frac{x}{c}\right)\right]. \tag{3}$$

The same expression can be obtained directly in first order of perturbation theory. An important feature of formula (3) is the periodic dependence of the depth of modulation on z with a period $\ell = (\pi v/\omega)(4E_0/\hbar\omega) = 3.2$ cm. This periodicity, which has essentially a quantum character, has not been considered previously. The dependence of the depth of modulation on z was not investigated in [1].

Formula (3) does not take into account the bounded character of the electron beam in the xy plane. In accordance with (2), in the case of absorption (emission) of a photon, the electron acquires an x-component of the momentum equal to $\hbar k$ (or $-\hbar k$). This causes the wave packet corresponding to different n in formula (2) to propagate in different directions ($\theta = \hbar k/p_0 \sim 10^{-5}$). In the case of a bounded beam with diameter D, the wave packets with $n = 0$ and $n = 1$ overlap only when $z < D\theta$, and at larger values of z there is no interference between them and the modulation disappears. As seen from the figure, at $D\theta/2 < z < D\theta$ there exist two modulation regions, so that as the metallic screen is moved farther away, the glowing spot begins to split in two and then vanishes. On a luminescent screen, there appear then three individual spots, one bright and two weaker ones on the edges.

5. At large values of the parameter $u/\hbar\omega$ modulation appears at higher harmonics. If the incoming electron is described by a plane wave, the current density is given by

$$j(r\ t) = ev\left[1 + 2\cdot\sum_{s=1}^{\infty} J_s\left(\frac{2u}{\hbar\omega}\sin\pi\frac{sz}{\ell}\right)\sin s\omega\left(t - \frac{z}{v} - \frac{x}{c}\right)\right]. \tag{4}$$

The expression for the amplitude of the s-th harmonic in (4) is valid when $sz \ll (2/3)(\hbar\omega E_0/u^2)\ell$. The transition to the classical limit occurs at $sz \ll \ell$. When $sz \gg \ell_{cl} = (\hbar\omega/u)\ell$, the modulation-depth decrease known in klystron theory [3] takes place. The remarkable difference between the quantum formula (4) and the classical formula is that at a distance $z \sim \ell/s \gg \ell_{cl}/s$ there appears again a high degree of modulation, and the regions in which the modulation is large repeat periodically in space, with a period ℓ/s. Allowance for the bounded character of the electron beam leads at $u/\hbar\omega \gg 1$ to a complicated z-dependence of the pattern of the light spots on the screen.

6. The radiation produced when a modulated beam of electrons interacts with a screen was considered in [2]. There, however, no account was taken of the dependence of the phase of the modulation on the coordinate x (cf. with (4)). Without presenting here the details of the characteristics of the

radiation, we note only that its angular distribution is strongly anisotropic and depends on the orientation of the screen. In the experiment [1], such an anisotropy was not observed apparently, since the surface of the employed screen was not flat with an accuracy to within the wavelength of the light.

In conclusion, we call attention to the need for further experimental study of the modulation of an electron beam at optical frequencies. It would be quite interesting to observe the aforementioned quantum singularities of this phenomenon.

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POSSIBILITY OF STABILIZING FLUTE INSTABILITY OF A PLASMA WITH THE AID OF CONTROLLED ELECTRON BEAMS

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Experiments have shown [1, 2] that the first mode of flute instability of a plasma in a trap with a straight mirror field can be suppressed with the aid of external electrodes, the potentials of which are controlled by a feedback system in accordance with the fluctuations on the surface of the plasma [3]. Such a stabilization system, however, cannot act in principle on the internal modes, which are not sensitive to the boundary conditions. To stabilize such oscillations, it is necessary to find a method of introducing the feedback signal directly into the volume of the plasma. Since in flute oscillations the phase of the perturbation is constant on the force lines of the magnetic field, it is possible to use for this purpose a system of electron beams injected into the plasma from the space outside the mirrors. The intensity of these beams should change in accordance with the fluctuations of the potential on the corresponding force lines. (Methods of stabilizing drift oscillations with the aid of controlled sources of electrons placed inside the plasma and with the aid of beams of neutral atoms were considered in [4, 5].)

The problem of flute oscillations of a rarefied plasma in a strong magnetic field was considered many times. Our case differs only in the fact that it is necessary to take into account the change of electron-beam density in the continuity equation for the electrons, i.e., to introduce a term of the form $(1/ev_b)(\partial j/\partial t)$, where v_b is the velocity of the electrons in the beam, e is the electron charge, and j is the density of the electron current. Since we are particularly interested in stabilization of short-wave oscillations, it is convenient to carry out the analysis in the quasiclassical approximation. Let a cold plasma, homogeneous in the directions of y and z, be placed in a strong magnetic field parallel to the z axis. Let the plasma density decrease in the direction of the x axis. To simulate the effect of curvature of the force lines, we introduce a gravitational force directed along the x axis and leading to drift of the ions along the y axis with velocity v*. Then, for small potential perturbations of the form $f_0 \exp i(k_x x + k_y y = \omega t)$, the dispersion equation will take the form (cf., e.g., [6]):

$$-(k_x^2 + k_y^2)\left(1 + \frac{\omega_{0i}^2}{\omega_{Bi}^2}\right) - \frac{i4\pi}{\omega v_b}\left(\frac{\partial j/\partial t}{f}\right) = \frac{\omega_{0i}^2}{\omega_{Bi}}\kappa k_y\left(\frac{1}{\omega} - \frac{1}{\omega + k_y v^*}\right), \qquad (1)$$