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INTERACTION OF NEUTRINO WITH NUCLEI AT HIGH ENERGIES

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In [1], Gribov considered the scattering of high-energy electrons by heavy nuclei, assuming that large distances x are involved in the integral (1) for the amplitude of the forward Compton effect

$$F_{\mu\mu} = i\int e^{iqx} < A|\, T j_\mu\left(-\frac{x}{2}\right) j_\mu\left(\frac{x}{2}\right) |A> d^4x\,, \qquad (1)$$

i.e., the process proceeds in two stages. At first the photon decays into virtual hadrons, and then these hadrons interact with the nucleus.

Let us consider, under the same assumptions, the interaction between high-energy neutrinos and nuclei. Neglecting the lepton mass, we obtain [2]

$$\frac{d\sigma}{dq^2 d\nu} = \frac{k_{20}G^2}{k_{10}2\pi}\left[\cos^2\frac{\theta}{2}\, W_2(\nu, q^2) + 2\sin^2\frac{\theta}{2}\, W_1(\nu, q^2) + \right. \qquad (2)$$
$$\left. + \frac{k_{10}+k_{20}}{M}\sin^2\frac{\theta}{2}\, W_3(\nu, q^2)\right],$$

$$M_{\mu\nu}(\nu, q^2) = \int \sum_\beta < A|l_\mu^+|\beta> < \beta|l_\nu^-|A> (2\pi)^3\delta^4(k_\beta + k_2 - k_1 - p)\,\times$$
$$\times\frac{d^3k_\beta}{4k_{\beta 0}M(2\pi)^3} = -\delta_{\mu\nu}W_1 + \frac{p_\mu p_\nu}{M^2}W_2 + i\,\epsilon_{\mu\nu k\ell}p_k q_\ell\frac{W_3}{M^2} + \frac{p_\mu q_\nu + q_\mu p_\nu}{M^2}W_4 \qquad (3)$$
$$+ \frac{q_\mu q_\nu}{M^2}W_5\,.$$

Here k_1 and k_2 are the 4-momenta of the incident and scattered leptons, p is the 4-momentum of the nucleus, M is the nuclear mass, θ is the scattering angle in the lab system, and

$$q = k_1 - k_2\,,\quad \nu = q_0 = (pq)/M\,.$$

Allowance for only large distances is equivalent [3, 1] to postulating nonsubtractive dispersion relations in terms of the mass q^2 for the amplitudes $\langle A | I_\mu^+ | \beta \rangle$

$$\langle A | I_\mu^+ | \beta \rangle = \frac{1}{\pi} \int \sum_n \frac{\langle 0 | I_\mu^+ | n \rangle \langle n A | \beta \rangle}{q_n^2 - q^2} \, dq_n^2 \, . \tag{4}$$

To sum over the states $|\beta\rangle$ we use the unitarity condition

$$\sum_\beta \langle nA | \beta \rangle \langle \beta | mA \rangle = -2 Im(i \langle nA | mA \rangle) = 2 \mathrm{Im} F_{nm} . \tag{5}$$

The amplitude F_{nm} for the scattering of fast hadrons by a nucleus was calculated by Gribov [1]

$$F_{nm} = i2 |\mathbf{q}| M \left[2\pi R^2 \delta(n-m) + 0\left(\frac{(M_b^2 - q_n^2)^2}{4\nu^2} \ell^2 \right) \right] \tag{6}$$

Here ℓ is the mean free path of the hadrons in the nucleus, and it is assumed that the nuclear radius $R >> \ell$. But since formula (6) is approximate, it is impossible to determine W_2 and W_4 from the dispersion relations.

Let us consider a conserved vector current. Then

$$W_4^v = \frac{M^2 W_1^v - q^2 W_5^v}{pq} ; \qquad W_2^v = - \frac{q^2}{pq} W_4^v \tag{7}$$

and writing down the non-subtractive dispersion relations for W_1^V and W_5^V, we obtain

$$W_1^v(\nu, q^2) = \nu R^2 \int \frac{M^2 \rho_1^v (M^2) dM^2}{(M^2 - q^2)^2} ; \quad W_2^v = \frac{-q^2}{\nu} R^2 \int \frac{\rho_1^v(M^2) \, dM^2}{(M^2 - q^2)} , \tag{8}$$

where

$$\sum_n \langle 0 | I_\mu^v | n \rangle \langle n | I_\nu^v | 0 \rangle (2\pi)^4 \delta(k_n - q) = -(\delta_{\mu\nu} q^2 - q_\mu q_\nu) \rho_1^v(q^2). \tag{9}$$

It turns out here that $W_2 \to$ const as $q^2 \to \infty$. Consequently [3] W_2 involves small distances that are uniquely connected with the large ones for W_5 by virtue of the current conservation.

Let us estimate the total neutrino absorption cross section, taking into account only the vector current. Integrating (2) with respect to ν and q^2 in the limits $0 < \nu < E$, $0 < q^2 < \gamma\nu M$, and $0 < \gamma << 1$, and substituting expression (8) for W_2^V, we obtain

$$\sigma > \frac{G^2}{4\pi} M E \gamma R^2 \int dM^2 \rho_1^v (M^2) . \tag{10}$$

A similar growth of the cross section with increasing energy was obtained by Bjorken [4] by using current abgebra.

This result remains unchanged when the axial current is taken into account, since it follows from (3) that

$$W_{1,2} = W^A_{1,2} + W^v_{1,2}\,.\quad W^A_{1,2} > 0,\ W_2 > \frac{|q^2|}{\nu^2} W_1 > \frac{|q^2|}{\nu M} |W_3|$$

and the integration over the regions $0 < \nu < E/2$ and $(2/3)E < \nu < E$ yields

$$\sigma > \frac{G^2}{16\pi} M E \gamma R^2 \int \rho^v_1 (M^2)\, dM^2 .$$

It is easy to see that the subtraction constants in the dispersion relations can likewise not limit the growth of the cross section.

Indeed, by virtue of relations (7) and the positiveness of W_1, it is necessary to have $W_5 < 1/q^2$ as $q^2 \to \infty$ in order for the cross section not to increase with energy. But then non-subtractive dispersion relations are applicable to W_5 and formula (10) is valid.

We have thus shown that in spite of the allowance for strong interactions, the total cross section for the absorption of a neutrino by a nucleus does not increase faster than the first power of the energy. This is due to the fact that the probability of the decay into virtual particles decreases slowly with increasing q^2 (only because of the propagator $1/(M^2 - q^2)$).

The axial current is not conserved, but we can stipulate for it, in the spirit of the Nambu hypothesis, that $q_\mu \langle A|I_\mu|\beta\rangle \to 0$ as $q^2 \to \infty$, and we can write non-subtractive dispersion relations for W^A_1, W^A_5, W_3, $q_\mu M_{\mu\nu} p_\nu$, and $q_\mu M_{\mu\nu} q_\nu$. Then,

$$W^A_1 = \nu R^2 \int \frac{M^2 \rho^A_1 (M^2)}{(M^2 - q^2)^2}\, dM^2 , \qquad W_3 = 0.$$

$$W^A_2 = \frac{R^2}{\nu} \int dM^2 \left[\rho^A_2 (M^2) - \frac{q^2 \rho^A_1 (M^2)}{M^2 - q^2}\right], \tag{11}$$

where

$$\sum_n \langle 0|I^A_\mu|n\rangle \langle n|I^A_\nu|0\rangle (2\pi)^4 \delta(k_n - q) = -(\delta_{\mu\nu} q^2 - q_\mu q_\nu)\rho^A_1(q^2) + q_\mu q_\nu \rho^A_2(q^2) .$$

Just as in Gribov's paper [1], we obtained a cross section proportional to the number of nucleons on the surface of the nucleus, $A^{2/3}$. It was assumed here that the nucleus is heavy, $R \gg \ell$.

The dependence of the cross section on the dimensions of the nucleus, within the framework of the π-dominance model, was considered by Trefil [5].

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