When $d(d\phi/dn_h)/dn_h > 0$ or $d^2n_h/d\phi^2 < 0$, the slope of the wave front increases without limit.

Leaving aside for the time being the question of the width of the stationary "shock" wave, let us determine the velocity of this wave. From (1) and (2) we can obtain the conservation laws and the velocity of the jump

$$D = a v_o \sqrt{\frac{m}{3M}\frac{n_o}{n_{oh}}}, \quad \frac{mv_o^2}{2} = e\phi_{max}. \tag{4}$$

Here $v_0$ is the maximum velocity of the particles for the given distribution function.

The lower bound of the width of the jump, $(M/m)^{1/2}r_{de}$, can be obtained by stipulating that the spectrum of the ion-acoustic oscillations have time to become established within the jump.

As already noted, the condition for the formation of the "shock wave" imposes limitations on the $n(\phi)$ dependence, which is determined from the velocity distribution functions of the hot particles. Thus, for example, for a Maxwellian distribution $d(d\phi/dn_h)/dn_h < 0$ no stationary wave is formed, and the front spreads out in accordance with the self-similar solution (x/t). For distribution functions having a steeper decrease (for example, for a Maxwellian distribution in the form of a step), we have

$$n_h \sim \left(e\phi + \frac{mv_o^2}{2}\right)^{1/2}, \quad \frac{\partial^2 n_h}{\partial \phi^2} < 0$$

and a discontinuity is formed. In the general case, the distribution functions can lead to a complicated $n(\phi)$ dependence, when both steepening and spreading can occur on different sections of the front. In principle, the shape of the front can yield information concerning the form of the distribution function of the hot electrons. Thus, the propagation of heat in a collisionless plasma will be realized by the foregoing mechanism and can be accompanied by the formation of a wave with a steep front. A similar effect may turn out to be quite important for plasma heating by a powerful relativistic beam.

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## FACTORIZATION OF AN N-POINT DUAL AMPLITUDE

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Considerable progress was made recently in the study of the N-point Veneziano amplitudes $B_N$. Mandelstam [1] and Olesen [2, 3] have shown that $B_N$ can be factored if the intersections of the Regge trajectories depend bilinearly on the additive "quantum numbers" $b_i$ of the external particles.

The conditions for the compatibility of the connection between the trajectories and the external particles leads to conservation of these "quantum numbers," $\sum_{i=1}^{N} b_i = 0$, but makes it necessary to assign to identical external particles quantum numbers $b_i$ of both signs simultaneously.

We consider in this paper the factoring of the Veneziano amplitude for a specified finite set of trajectories. As usual, it is assumed that all the particles, both external and resonances, lie on these trajectories, which have a universal slope $\alpha'$.

Assume that we have $2^n$ trajectories, each of which is set in unique correspondence with a set of n "quantum numbers" $\{T^{\alpha}\}, T^{\alpha} = \pm 1$, $\alpha = 1, 2, \ldots, n$.

In order that the Regge trajectories in any of the dual channels (i, j) of the amplitude $B_N$ be uniquely determined from the "quantum numbers" of the external particles, it suffices to stipulate the satisfaction in each channel of the conditions

$$\prod_{k=i}^{j} T_k^{\alpha} = T_{(ij)}^{\alpha}, \tag{1}$$

where $\{T_{(ij)}^{\alpha}\}$ are the "quantum numbers" corresponding to the Regge trajectory in the channel (i, j).

We denote by $G^{\mu}$ ($\mu = 5, 6, \ldots, 2^n + 3$) the non-coinciding products of the "quantum numbers" $T^{\alpha}$ of the Regge trajectory, and put $G^{\mu} = 1$ at $\mu = 1, 2, 3, 4$. Then, in the general case, the intersection for any Regge trajectory in the (i, j) channel is written in the form

$$A_{ij} = \sum_{\mu=4}^{2^n+3} \beta^{\mu} \prod_{k=i}^{j} G_k^{\mu}, \tag{2}$$

where k are the indices of the external particles of the block. It is obvious that the $\beta^{\mu}$ are linear combinations of the intersections of $2^n$ (specified) different trajectories.

The amplitude can be factorized if the linear combinations of the intersections

$$C'_{k\ell} = A_{k\ell} + A_{k+1,\ell-1} - A_{k+1,\ell} - A_{k,\ell-1}$$

can be represented in the bilinear form

$$C'_{k\ell} = 2\alpha' \sum_{\mu=5}^{M} d_k^{\mu} d_{\ell}^{\mu},$$

where $d_k^{\mu}$ and $d_{\ell}^{\mu}$ pertain to blocks I and II in the figure, respectively.

For the $A_{k\ell}$ intersections in the form (2) we have

$$C'_{k\ell} = \sum_{\mu=5}^{2^n+3} \beta^{\mu} \prod_{s=k+1}^{\ell-1} G_s^{\mu} (1 - G_k^{\mu})(1 - G_{\ell}^{\mu}). \tag{3}$$

We introduce now $(2^n - 1)$-dimensional "momenta"

$$p^\mu = \sqrt{\beta^\mu}(1 - G^\mu) \qquad \mu = 5, 6, \ldots, (2^n + 3). \tag{4}$$

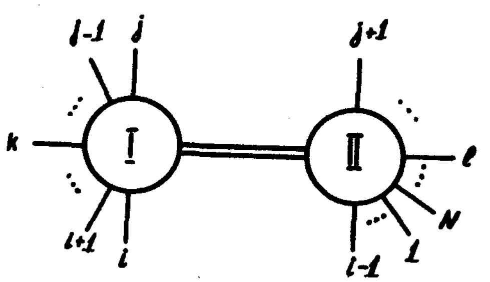
The exponents in the Bardakci-Ruegg formula for the amplitude $B_N$ take the form (A - intersection of trajectory for which all $G^\mu = 1$):

$$\gamma_{k\ell} = -\sum_{\mu=1}^{4} p_k^\mu p_\ell^\mu - \sum_{\mu=5}^{2^n+3} \left(p_k^\mu \prod_{s=k+1}^{j} G_s^\mu\right)\left(p_\ell^\mu \prod_{t=j+1}^{\ell-1} G_t^\mu\right) - \delta_{k,\ell-1}(2 - A) \tag{5}$$

and obviously the amplitude $B_N$ is completely factorized. It is clear that in this case the degree of degeneracy of the levels $\alpha(s) = J$ depends on the number of "quantum numbers" $\{T^\alpha\}$. The asymptotic level density is

$$d(J) \sim \exp\left[\frac{2\pi}{\sqrt{6}} \sqrt{(2^n + 3)J}\right].$$

It is easy to generalize the Fubini-Gordon-Veneziano operator formalism [4] to the foregoing case of $2^n$ different Regge trajectories. Besides the usual operators $a_{r,\mu}$ ($\mu = 1, 2, 3, 4$) we introduce additional $(2^n - 1)$ sets of operators $a_{r,\mu}$ ($\mu = 5, 6, \ldots, 2^n - 1$) corresponding to the additional components of the momentum $p^\mu$. Then the vertex operators and the propagators are written in the form




$$\hat V_{ijk} = \theta_{ijk} \exp\left[p_j \sum_{r=1}^{\infty} \frac{a_k^+}{\sqrt{r}}\right] \hat Q_j \exp\left[p_j \sum_{r=1}^{\infty} \frac{a_r}{\sqrt{r}}\right],$$

$$\hat D_{\ell m} = \prod_{\mu=1}^{2^n+3} \delta[G_\ell^\mu, G_m^\mu] \int_0^1 du_\ell\, u_\ell^{\hat R_\ell} (1 - u_\ell)^{A-2},$$

where

$$\theta_{ijk} = \frac{1}{2}\left(1 + \prod_{\alpha=1}^{n} T_i^\alpha T_j^\alpha T_k^\alpha\right),$$

$$\hat Q_j = \prod_{\mu=1}^{2^n+3} (G_j^\mu)^{h_\mu}; \qquad h_\mu = \sum_{r=1}^{\infty} a_{r,\mu}^+ a_{r,\mu},$$

$$\delta[G_\ell^\mu; G_k^\mu] = \begin{cases} 1 & G_\ell^\mu = G_k^\mu \\ 0 & G_\ell^\mu \neq G_k^\mu \end{cases},$$

$$\hat R_\ell = -A - \frac{1}{2} p_\ell^2 + \sum_{r=1}^{\infty} r\, a_{r,\mu}^+ a_{r,\mu}$$

$$p_j a_r = \sum_{\mu=1}^{2^n+3} p_j^{\mu} a_{r,\mu} .$$

The factor $\theta_{ijk}$ in the vertex operator ensures satisfaction of the condition (1) at each vertex. The need for introducing the factor $Q_j$ is dictated by the presence of the product

$$\prod_{s=k+1}^{\ell-1} G_s^{\mu}$$

in the second term of the right side of (5): these factors arise automatically in $\gamma_{\ell k}$ when the operators $\hat{V}\hat{D}\hat{V}...\hat{D}\hat{V}$ are reduced to the normal product.

Thus, in the case considered by us the Veneziano amplitude $B_N$ can be factorized for any N for a finite number ($2^n$) of different main trajectories, in contrast to the case considered by Olesen [3], where the spectrum of the main trajectories is infinite.

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## ANNULAR TRAP FOR LOW-FREQUENCY WAVE IN THE EARTH'S MAGNETOSPHERE

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1. The distribution of high-energy particles in the plasma surrounding the earth is usually unstable. This leads to self-excitation of different types of electromagnetic waves. For example, as a result of cyclotron instability of the protons of the radiation belt, Alfven waves are excited and are observed on the earth in the form of geomagnetic pulsations in the region of ~1 Hz (see the review [1]).

It is important that the instability has as a rule a convective character. The generation arises therefore, for example, in those cases when the wave packet has an opportunity of crossing many times the region of interaction with the resonant particles. For Alfven waves, such a possibility is afforded by the magnetic focusing and reflection of the waves from the ionosphere on opposite ends of the force tube.

Magnetosonic waves are not subject to magnetic focusing, so that their trajectories are rather complicated curves. In the general case these waves leave the resonant region rapidly. We shall attempt, however, to find conditions that permit a prolonged interaction between magnetic sound and high-energy particles.

2. We consider first rays lying in the plane of the geomagnetic equator (transverse propagation). Obviously, the necessary condition is that the curvature of the ray be equal to the curvature of the drift shell. It is easy