$$p_l a_r = \sum_{\mu=1}^{2^n+3} p_l^{\mu} a_{r,\mu} .$$

The factor θ_{ijk} in the vertex operator ensures satisfaction of the condition (1) at each vertex. The need for introducing the factor Q_j is dictated by the presence of the product

$$\prod_{s=k+1}^{\ell-1} G_s^{\mu}$$

in the second term of the right side of (5): these factors arise automatically in $\gamma_{\ell k}$ when the operators $\hat{V}\hat{D}\hat{V}...\hat{D}\hat{V}$ are reduced to the normal product.

Thus, in the case considered by us the Veneziano amplitude B_N can be factorized for any N for a finite number (2^n) of different main trajectories, in contrast to the case considered by Olesen [3], where the spectrum of the main trajectories is infinite.

The authors are sincerely grateful to Yu.K. Krasnov for useful discussions.

[1] S. Mandelstam, Berkeley Preprint UCRL-19327, 1969.
[2] P. Olesen, CERN Preprint TH-1100, 1969.
[3] P. Olesen, CERN Preprint TH-1131, 1970.
[4] S. Fubini, D. Gordon, and G. Veneziano, Phys. Lett. 29B, 679 (1969).

ANNULAR TRAP FOR LOW-FREQUENCY WAVE IN THE EARTH'S MAGNETOSPHERE

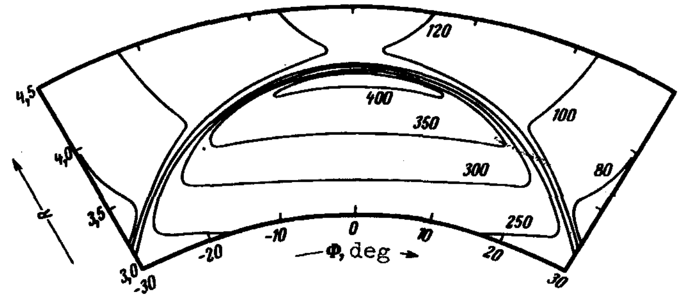
A.V. Gul'el'mi
Institute of Earth Physics, USSR Academy of Sciences
Submitted 20 May 1970
ZhETF Pis. Red. 12, No. 1, 35 - 37 (5 July 1970)

1. The distribution of high-energy particles in the plasma surrounding the earth is usually unstable. This leads to self-excitation of different types of electromagnetic waves. For example, as a result of cyclotron instability of the protons of the radiation belt, Alfven waves are excited and are observed on the earth in the form of geomagnetic pulsations in the region of $\sim$1 Hz (see the review [1]).

It is important that the instability has as a rule a convective character. The generation arises therefore, for example, in those cases when the wave packet has an opportunity of crossing many times the region of interaction with the resonant particles. For Alfven waves, such a possibility is afforded by the magnetic focusing and reflection of the waves from the ionosphere on opposite ends of the force tube.

Magnetosonic waves are not subject to magnetic focusing, so that their trajectories are rather complicated curves. In the general case these waves leave the resonant region rapidly. We shall attempt, however, to find conditions that permit a prolonged interaction between magnetic sound and high-energy particles.

2. We consider first rays lying in the plane of the geomagnetic equator (transverse propagation). Obviously, the necessary condition is that the curvature of the ray be equal to the curvature of the drift shell. It is easy



to verify that if the plasma density ρ decreases with increasing distance from the earth more slowly than R^{-8}, then the curvatures of the ray and of the magnetic shell have different signs, i.e., the foregoing condition is not fulfilled.

The required sharp decrease ρ across the magnetic shells is observed only in the region of the plasmapause [2, 3]. The plasmopause bounds from above the so-called plasmosphere (region of the dense plasma) and is the surface produced by revolution of the force line with L ∿ 4 around the geomagnetic axis (L is the distance from the earth's surface to the top of the line, in units of the earth's radius $r_e = 6.37 \times 10^8$ cm). The plasmopause thickness is $\sim 10^8$ cm.

The figure shows the isolines $n = c/A$ in the plane of the equator ($A = B/\sqrt{4\pi\rho}$ is the Alfven velocity, ϕ the latitude, and R the geocentric distance in units of r_e). It is seen from the structure of the figure that there exists under the dome of the plasmosphere a waveguide that channels the magnetic sound in the azimuthal direction (perpendicular to the plane of the figure). This channel, in the form of a ring with radius L ∿ 4, surrounds the earth.

In the flat channel model, the wave field is described by the equation[1])

$$\frac{d}{dz}\left[\frac{1}{n^2}\frac{db}{dz}\right] + \left\{\frac{\omega^2}{c^2} - \frac{\kappa^2}{n^2} + \frac{d}{dz}\left(\frac{\kappa g}{n^2 \epsilon}\right)\right\} b = 0 . \qquad (1)$$

The x axis is directed here along the channel axis and z is perpendicular to the plasmopause; $\nabla_x = i\kappa$, $\nabla_y = 0$, $\vec{b} = (b, 0, 0)$, $n^2 = \varepsilon - g^2/\varepsilon$, ε and g are defined as in [5]. Assuming that in the plasmosphere the properties of the medium vary little over one wavelength, we transform (1) into

$$\frac{d^2\Psi}{dz^2} + \left(\frac{\omega^2}{c^2} n^2(z) - \kappa^2\right)\Psi = 0, \qquad (2)$$

where $\Psi \equiv b/n$. At frequencies much lower than the hybrid frequency we have $n^2 = c^2/A^2$.[2]) At large wavelengths the plasmopause can be replaced by a sharp boundary, and at low wavelengths by a transition layer.

The magnetosonic channel recalls in its structure (which is determined by the function n(z)) an underwater acoustic channel [4] or the tropospheric radio waveguide [6]. Accordingly, the results of the solution of (2) given in [4, 6] can be used in this case with slight modification. Without dwelling on the details of the calculations, we note that the annular trap retains effectively waves having frequencies from fractions of one Hertz to several dozen Hertz in

[1])The curvature of the channel axis can be taken into account by introducing a modified refractive index [4].

[2])Dispersion effects will be observed near the hybrid-resonance frequency (∿300 Hz) and in narrow bands near the proton gyrofrequency (∿8 Hz) and the He^+ ion geofrequency (∿2 Hz).

a wide range of propagation directions.

3. The usual dissipation mechanism, wave absorption in the magnetic mirrors, does not exist in the magnetosonic channel. Partial leakage of the captured waves through the upper and lower walls of the channel is likewise excluded (total internal reflection). Therefore even a very slight supercriticality of the distribution of the high-energy protons leads to an exponential growth of the wave intensity.

In the case of almost transverse propagation, magnetic sound can be excited as a result of Cerenkov instability of the protons, with a nonmonotonic dependence of the distribution function on the energy [7]. In order of magnitude, the maximum increment is equal to

$$\gamma \sim \frac{N'}{N} \Omega_i \frac{W}{A} , \tag{3}$$

where N', W, and Ω_i are the concentration, average velocity, and the gyrofrequency of the high-energy protons, and N is the density of the cold plasma. The resonant frequency is $\omega \sim \Omega_i (A/W)$.

The energy of the magnetic sound in the channel will accumulate until the nonlinear processes come into play. Decays and coalescences of hydromagnetic wave packets limit the amplitude of the oscillations more effectively than the quasilinear processes. A tentative estimate yields the following expression for the amplitude of the steady-state oscillations:

$$b^2 \sim [\mathcal{E} N'/(B^2/8\pi)]^{1/2} p_e / \sqrt{\Delta} . \tag{4}$$

Here $\mathcal{E} = m_i W^2/2$, p_e is the electron pressure of the cold plasma, and Δ is the width of the channel in wavelength units. In the derivation of (4) it was assumed that the magnetosonic waves increase with the increment (3), and that the other partners in the nonlinear interaction, namely the Alfven and acoustic waves, attenuate. At the typical values $L \sim 4$, $\mathcal{E} \sim 50$ keV, $N' \sim 0.1$ cm^3, $p_e \sim 10^{-9}$ erg/cm^3, and $\Delta \sim 10$ we have $b \sim 1$ at a frequency ~ 1 Hz.

4. The hypothetical existence of a magnetosonic channel can be verified by direct observation of low frequency waves ($\sim 10^{-1} - 10^{-2}$ Hz) in the equatorial region of the plasmopause. Land-based observations are also promising, since scattering of the waves by the inhomogeneities of the channel, decay processes, and the like give rise to a wave energy flux to the earth at two conjugate latitude regions $\phi \sim \pm 60°$.

It is advisable to perform the appropriate experiments because hydromagnetic waves, as is well known, can be used as scattering centers for particles from the radiation belts. Since the plasmopause shifts slowly in the range $L \sim 3 - 6$, the waves accumulated in the annular trap act on the particles in the entire space of the outer radiation belt.

[1] V.A. Troitskaya and A.V. Gul'el'mi, Usp. Fiz. Nauk 97, 453 (1959) [Sov. Phys.-Usp. 12, 195 (1969)].
[2] K.I. Gringauz, Geofiz. byulleten' 14, 110 (1965).
[3] D.L. Carpenter, J. Geophys. Res. 71, 693 (1966).
[4] L.M. Brekhovskikh, Volny v sloistykh sredakh (Waves in Layered Media), M. 1957. [Academic Press, 1960].
[5] V.D. Shafranov, in: Voprosy teorii plazmy (Problems of Plasma Theory), M. A. Leontovich, ed., M., 3, 3 (1963).

[6] Ya.L. Al'pert, Rasprostranenie radiovoln i ionosfera (Radio Wave Propagation and the Ionosphere), M., 1960. [Consultants Bureau, 1963].
[7] L.V. Korablev, L.I. Rudakov, Zh. Eksp. Teor. Fiz. 54, 818 (1968) [Sov. Phys.-JETP 27, 439 (1968)].

IMPORTANCE OF MEASURING THE MAGNETIC ANISOTROPY IN THULIUM

Yu.P. Irkhin
Metal Physics Institute, USSR Academy of Sciences, Siberian Division
Submitted 22 May 1970
ZhETF Pis. Red. 12, No. 1, 38 - 40 (5 July 1970)

The construction of a microscopic theory of the magnetic anisotropy of magnetic crystals entails considerable difficulties. In rare-earth ferromagnets, however, where the magnetic f-electrons are well described by atomic functions localized at the lattice sites, the problem is greatly facilitated. An important consequence of this localization is the fact that the orbital momenta of the f-electrons remain unquenched and form, together with the spin momenta, total angular momenta $\vec{J}$ in accordance with the Russel-Saunders scheme[1]. The existence of unquenched orbital momenta rigidly coupled with the spin momenta greatly facilitates the analysis of the magnetic anisotropy, for now the angular momentum $\vec{J}$ becomes directly oriented along definite crystallographic axes. In d-magnets, the effect of crystallographic orientation of the spin momenta is attained only in the second approximation in the spin-orbit interaction $\lambda(\vec{L}\vec{S})$, since the first approximation vanishes as a result of the quenching ($L_z = 0$).

Even this qualitative reasoning explains the appreciably larger magnetic anisotropy of rare earths compared with d-magnets. More detailed results can be obtained by analyzing different anisotropic contributions to the energy of the f-electrons in the crystal. It turns out here that for certain main contributions it is possible to obtain the theoretical dependence of the corresponding terms of the Hamiltonian on the number of the element (more accurately, on the quantum numbers S, L, and J of the lowest term of the configuration f^n). By calculating subsequently some macroscopic quantities, we can compare these relations with the experimental data.

We have already presented [1] results on the anisotropy of the paramagnetic susceptibility of rare-earth metals for the anisotropic-exchange mechanism. In the present paper we shall show that it is possible to separate the two main mechanisms (crystal-field and the anisotropic-exchange) of magnetic anisotropy by calculating the magnetic-anisotropy constant K_1.[2]

It is simplest to consider the crystal-field mechanism. The second-order anisotropy energy can be written in the form

$$E_a^{(2)} = 2\alpha_J A_2 J^2 P_2(\cos\theta). \qquad (1)$$

Here A_2 is an energy parameter that determines the splitting in the crystal field, α_J is a certain coefficient tabulated, for example, in [3], and P_2 is a Legendre polynomial.

---

[1])Conversely, in magnets of the iron group, the strong crystalline field smears out the atomic levels of the d-electrons into bands, quenching by the same token their orbital momenta.

[2])These results were reported in part at the Tbilisi Conference on Low Temperatures in 1968 [2].