the crystal-field theory the absolute values of K_1 for Tb and Tm are equal[3], whereas according to the anisotropic-exchange theory they differ by a factor of 3.

A similar situation obtains also for the anisotropy of the paramagnetic susceptibility [1], but experimental data on this quantity are again lacking in the case of Tm.[4]

Of course, a noticeable difference in the predicted anisotropy occurs also for Er, but owing to the low accuracy of both the experimental and the theoretical estimates, measurements of the anisotropy in Tm would be of greatest interest at the present time.

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INTERACTION OF HADRONS WITH DEUTERONS AT HIGH ENERGIES, INELASTIC PROCESSES AND DUALITY

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The purpose of the present paper is to trace qualitatively the energy behavior of the screening correction to the total cross section for the interaction between a high-energy hadron (henceforth, a pion) with a deuteron, with allowance for the inelastic processes.

The correction to the impulse approximation for the forward πd-scattering amplitude is given by [1]:

$$\delta F(s) = \frac{1}{m} \int \frac{dq}{(2\pi)^3} A(p_\pi,\ p_d, q)\rho(q), \tag{1}$$

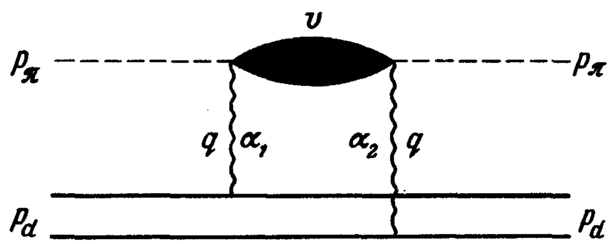

Fig. 1

[3] We emphasize that this equality is exact within the framework of the crystal-field theory.

[4] We note that no term of the type qnD_2 (formula (10) of [1]) arises for conduction electrons within the framework of the isotropic model considered there, by virtue of which the results for $\Delta\theta$ are somewhat altered.

where m is the nucleon mass, $\rho(\vec{q})$ is the deuteron form factor, and $A(\vec{p}_\pi, \vec{p}_d, \vec{q})$ is the connected part of the amplitude of forward scattering of a pion by the two free nucleons contained in the deuteron.

Figure 1 shows the diagram corresponding to $A(\vec{p}_\pi, \vec{p}_d, \vec{q})$ in the case when the interaction is described by exchange of reggeons α_1 and α_2 with signatures θ_{α_1} and θ_{α_2}:

$$A(p_\pi, p_d, q) = \sum_{\alpha_1\alpha_2} g_{\alpha_1}(\tau)\left(\frac{s}{s_0}\right)^{\alpha_1(\tau)+\alpha_2(\tau)} \times \theta_{\alpha_1}(\tau)\, g_{\alpha_2}(\tau)\, \theta_{\alpha_2}(\tau)\, U_{\alpha_1\alpha_2}(v,\tau). \tag{2}$$

Here $v = (p_\pi + q)^2$ is the square of the mass of the beam produced upon rescattering, $\tau \equiv \vec{q}_\perp^{\,2}$, $s_0 \simeq m^2$; $U_{\alpha_1\alpha_2}(v, \tau)$ is the amplitude of the transition $\alpha_1 + \pi \to \alpha_2 + \pi$, which has at large v the following Regge behavior [2]:

$$U_{\alpha_1\alpha_2}(v,\tau) = s_0 \sum_\beta r^\beta_{\alpha_1\alpha_2}(\tau)\, g_\beta(0) \left(\frac{v}{s_0}\right)^{\beta(0)-\alpha_1(\tau)-\alpha_2(\tau)}, \tag{3}$$

where $r^\beta_{\alpha_1\alpha_2}(\tau)$ is the vertex of the coupling of the reggeon β with the reggeons α_1 and α_2.

We assume [3] that (3) determines the behavior of $U_{\alpha_1\alpha_2}$ averaged over v also at small values of v. Since $U_{\alpha_1\alpha_2}(v, \tau)$ enters in (1) under the sign of an integral with respect to v, such a utilization of duality "in the mean" [4] for Im $U_{\alpha_1\alpha_2}$ is reasonable.

Choosing for $\rho(\vec{q})$ the Gaussian form $\exp(-R^2\vec{q}^{\,2})$, where R is the radius of deuteron, we obtain from (1) - (3), after integrating with respect to τ and making a number of transformation, the following expression for the correction to $\sigma^+_{\pi d}$ for the double rescatterings:

$$\delta\sigma(s) = \sum_{\alpha_1\alpha_2\beta} \lambda^\beta_{\alpha_1\alpha_2}\left(\frac{s}{s_0}\right)^{\alpha_1(0)+\alpha_2(0)-2} \times \int_0^\infty \frac{d\eta \exp\left\{\eta\left(1+\beta(0)-\alpha_1(0)-\alpha_2(0)-\dfrac{R^2m^2(v-m^2)^2}{s^2}\right)\right\}}{[R^2+(\alpha_1'+\alpha_2')(\xi-\eta)+\nu_{\alpha_1\alpha_2\beta}]^C}, \tag{4}$$

where $\eta = \ln v/s_0$, $\xi = \ln s/s_0$, and the constants $\lambda^\beta_{\alpha_1\alpha_2}$ result from the following parametrization of the quantity

$$\Lambda^\beta_{\alpha_1\alpha_2} = \frac{1}{4\pi^2}\,\mathrm{Re}[\theta_{\alpha_1}(\tau)\,\theta_{\alpha_2}(\tau)]\,\mathrm{Im}\,\theta_\beta(0)\, r^\beta_{\alpha_1\alpha_2}(\tau)\, g^{(\pi)}_\beta(0)\, g^{(n)}_{\alpha_1}(\tau)\, g^{(n)}_{\alpha_2}(\tau), \tag{5a}$$

$$\Lambda^\beta_{\alpha_1\alpha_2}(\tau) = \begin{cases} \lambda^\beta_{\alpha_1\alpha_2}\, e^{-\nu_{\alpha_1\alpha_2\beta}\tau} & \text{if } r^\beta_{\alpha_1\alpha_2}(0) \neq 0; \quad C = 1 \\ \tilde{\lambda}^\beta_{\alpha_1\alpha_2}\,\tau\, e^{-\nu_{\alpha_1\alpha_2\beta}\tau} & \text{if } r^\beta_{\alpha_1\alpha_2}(0) = 0; \quad C = 2 \end{cases} \tag{5b}$$

which describes the exponential τ-dependence of the vertex $r^{\beta}_{\alpha_1\alpha_2}$ and of the vertices g of the interaction of the particles with the reggeons.

It is necessary to distinguish in (4) between contributions of the following types:

$$\begin{pmatrix} & \beta = P & \\ \alpha_1 = P, & & \alpha_2 = P \end{pmatrix}, \begin{pmatrix} & f & \\ P & & P \end{pmatrix} \begin{pmatrix} & P & \\ P & & f \end{pmatrix}, \begin{pmatrix} & P & \\ f(\omega) & & f(\omega) \end{pmatrix}, \begin{pmatrix} & f(\rho) & \\ P & & f(\rho) \end{pmatrix}, \begin{pmatrix} & f(\rho) & \\ f & & f(\rho) \end{pmatrix},$$

where $\alpha_P(0) = 1$ and $\alpha_f(0) = \alpha_\rho(0) = \alpha_\omega(0) = 0.5$. Their analysis, in accord with (4) and (5a, b), leads to the following results:

1. $\binom{P}{PP}$ contributions. Here, as is well known [2], $r^P_{PP}(0) = 0$. Writing $r^P_{PP}(\tau)$ in the form $2\alpha_P' a\tau$, we get:

$$\delta\sigma(s)^P_{PP} = \tilde{\lambda}^P_{PP} \frac{\ln(1 + s/Rm^3)}{\left[R^2 - 2\alpha_P'\ln\left(\frac{m^2}{s} + \frac{1}{mR}\right) + \nu_{PPP}\right](R^2 + 2\alpha_P'\xi + \nu_{PPP})}, \tag{6}$$

where

$$\tilde{\lambda}^P_{PP} = -a\,\alpha_P' g^{(\pi)}_P(0)\, g^{(n)2}_P(0)\,/\,2\pi^2.$$

When $\alpha_P'\xi \ll R^2$, this contribution is small ($\sim 1/R^4$), when $\alpha_P'\xi \sim R^2$ it is of the order of $1/R^2$, and when $\xi \to \infty$ it tends to a constant value

$$\delta\sigma(\infty)^P_{PP} = -a\, g^{(\pi)}_P(0)\, g^{(n)2}_P(0)\,/\,4\pi^2(R^2 + \nu_{PPP}). \tag{7}$$

If $r^P_{PP}(0) \neq 0$, then we would have at $\alpha_P'\xi \gg R^2$ a logarithmic growth of $\delta\sigma(s)^P_{PP}$, leading at $\sigma^+_{\pi n}$ = const to a negative $\sigma^+_{\pi d}$.

2. The $\binom{f}{PP}$ term contains, in accord with duality, a contribution from the resonances in the v-channel [5], and in particular an optical contribution [6]. From (4) and (5a, b) we obtain:

$$\delta\sigma(s)^f_{PP} = \frac{-g^{(\pi)2}_P(0)\, g^{(n)2}_P(0)\,/\,4\pi + 4\lambda^f_{PP}\,\phi(Rm^3/s)}{R^2 + 2\alpha_P'\xi}, \tag{8}$$

where

$$\lambda^f_{PP} = -\frac{1}{4\pi^2} g^{(\pi)}_f(0)\, g^{(n)2}_P(0)\, r^f_{PP}(0),$$

$$\phi(x) = \begin{cases} 1 - x^{1/2}\Gamma(3/4), & x \ll 1 \\ \sqrt{\pi}/4x, & x \gg 1 \end{cases}.$$

3. The $\binom{P}{Pf}$ contribution:

$$\delta\sigma(s)^{P}_{Pf} \approx \frac{\lambda^{P}_{Pf}\,\Gamma(1/4)\,\tilde{\phi}(Pm^3/s)}{2(mR)^{1/2}[\,R^2 + (\alpha'_P + \alpha'_f)\ln mP + \nu_{Pf\,P}]}, \qquad (9)$$

where

$$\lambda^{P}_{Pf} = -\frac{1}{4\pi^2}\, g_f^{(\pi)}(0)\, g_P^{(\pi)}(0)\, g_P^{(n)}(0)\left[r^{P}_{Pf}(0) + r^{P}_{fP}(0)\right],$$

$$\phi(z) = \frac{2}{\Gamma(1/4)}\int_0^\infty dx\, e^{-x^2}(x+z)^{-1/2} \to 1 \quad \text{if } z \approx 0.$$

4. The $({}^{P}_{f(\omega)f(\omega)})$ contribution is small, since it can be readily verified that $\lambda^{P}_{f(\omega)f(\omega)} \simeq 0$ when $\alpha_{f(\omega)} \simeq 0.5$.

5. The $({}^{f(\rho)}_{Pf(\rho)})$ contribution decreases like $s^{-1/2}$ when $s \gg Rm^3$.

6. A similar situation obtains with the $({}^{f(\rho)}_{Pf(\rho)})$ contribution.

At small values of s/Rm^3 only the optical contributions matter in the last two terms, and at $s/Rm^3 \sim 1$ the contributions from the lower-lying resonances are also significant.

We see thus that when $s/m^2 \ll mR$ only the optical contribution is of importance in $\delta\sigma(s)$. When s increases, contributions 2 - 5 begin to grow rapidly and reach a maximum at $s \sim Rm^3$. With further increase of s, the contributions 4 and 5 decrease rapidly, while 3 remains constant. When $s \gg Rm^3$ the contributions 1 and 2 taken together vary logarithmically with s. The character of this variation is determined by the parameter

$$b = a g_P^{(\pi)}(0) \,/\, [\pi g_P^{(\pi)}(0) + 4 g_f^{(\pi)}(0)\, r^{f}_{PP}(0)\,].$$

When $b \gtrless 1$ we have a logarithmic growth (decrease). When $\xi \sim (mR)^2$ the contributions 1 and 2 become equal, and when $\xi \gg (mR)^2$ only the asymptotic contributions 1 and 2 remain in $\delta\sigma(s)$ (see (7) and (9)).

Figure 2 shows the schematic behavior of $-\delta\sigma(s)$ as a function of $s(\xi)$. Its different section can be interpreted as follows in terms of the Gribov correction [1] corresponding to contributions of type 1 and 2.

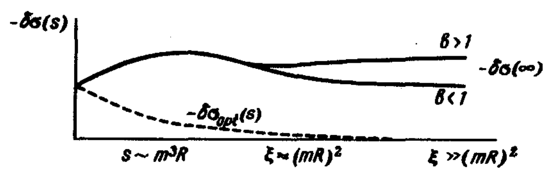

Fig. 2

The initial section of the curve ($s \lesssim Rm^3$) corresponds to inclusion of an ever increasing number of states in the integral $\int(\partial\sigma/\partial\vec{q}^{\,2})\,\rho\,(\vec{q}^{\,2})d\vec{q}^{\,2}$, namely resonances in the v-channel (terms 2, 5, 6) and the "v-channel interference background" (terms 3). The region of logarithmic variation of $\delta\sigma(s)$ ($\ln Rm < \xi < (mR)^2$) corresponds to a dropping-out of the contributions from the resonances (term 2) from the integral, owing to the contraction of the cones and to the logarithmic growth of the contribution of the diffraction background (term 1). Finally, when $\alpha'\xi \gg R^2$ the contributions of all the resonances drop out, and the background contributions 1 and 3, which determine $-\delta\sigma(\infty)$, dominate in $\partial\sigma/\partial\vec{q}^{\,2}$.

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SURFACE STATES IN SEMICONDUCTORS WITH COMPLICATED BAND STRUCTURE

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In semiconductors with complicated band structure, such as GaAs, there are sections of the spectrum in which the electrons have negative effective masses. This means that the repulsion potential can lead to the formation of bound states for the electrons.

We are interested in the interface between two media with different dielectric constants, particularly the semiconductor-vacuum interface. In this case, as is well known [1], the electron in the semiconductor is acted upon by an electrostatic image force that repels it from the surface. The potential energy of the electron is of the form α/z, where $\alpha = (e^2/4\kappa)(\kappa - 1)/(\kappa + 1)$, κ is the dielectric constant of the semiconductor, and z is the distance to the surface.

We assume that the usual conditions for the applicability of the equivalent-Hamiltonian approximation are satisfied (cf., e.g., [2]). The equation for the envelope $\Psi(\vec{r})$ in the expansion of the wave function of the electron in Wannier functions takes the form ($\hbar = 1$)

$$[\epsilon(-i\nabla) + \frac{a}{z}]\,\Psi(\mathbf{r}) = E\,\Psi(\mathbf{r}) \quad , \tag{1}$$

where $\varepsilon(\vec{p})$ is the law of the electron dispersion in the band. Since the potential energy depends only on z, the problem reduces to a one-dimensional one, i.e.,

$$\Psi(\mathbf{r}) = \phi(z)\exp\{ip_x x + ip_y y\} \ ,$$

where $\phi(z)$ satisfies the equation

$$\left[\epsilon(-i\frac{\partial}{\partial z}(p_x, p_y) + \frac{a}{z}\right]\phi(z) = E\,\phi(z). \tag{2}$$

Since $\phi(z)$ is an envelope function and its characteristic dimension should be much larger than the lattice period within the framework of the equivalent-Hamiltonian approximation the boundary condition for $\phi(z)$ is that it vanish at z = 0. It turns out that this problem has an exact solution at an arbitrary form of the dispersion $\varepsilon(\vec{p})$. We shall seek a solution in the form (for the k-th discrete level)

$$\phi(z) = e^{-\gamma_k z}(A_k z^k + A_{k-1}z^{k-1} + ... + A_1 z). \tag{3}$$