CONCERNING THE SECOND BRANCH OF THE SPECTRUM OF ELEMENTARY EXCITATIONS IN LIQUID He^4

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Recent reports of experimental observation of a second branch of elementary excitations in liquid helium [1] invite a discussion of this question from the theoretical point of view. We are concerned first with the possible existence of a bound state of two rotons with energy $\varepsilon < 2\Delta$. Such a bound state should be manifest in the form of a pole in the vertex part for roton-roton scattering. Such a vertex part was calculated in [2] at a summary roton energy $\varepsilon \approx 2\Delta$ and amounts to[1]

$$\gamma_2 = \frac{Q}{1 + Q \ln \frac{\alpha}{2\Delta - \epsilon}}, \quad \alpha(p) \to 0 \text{ as } p \to 2p_0. \tag{1}$$

(See formula (33) of [2]. The expression for γ_2 differs from the expression given in (33) for the "triple" vertex Γ only by a factor.) The subsequent analysis in [2] was based on the assumption that $Q > 0$. If, however, $Q < 0$, i.e., if the rotons are attracted, then the vertex will have a pole, at any arbitrarily small value of the interaction amplitude Q, at an energy

$$\epsilon_2(p) = 2\Delta - \alpha \exp(-1/|Q|) \tag{2}$$

and a new branch of the spectrum will appear in helium, with energy lower than 2Δ but close to it. It is easy to see that this branch should terminate undamped on the small-momentum side at $p \approx 0.95$ Å^{-1}. At this point, the two-particle excitation can break up into two single-particle excitations of energy $\sim\Delta$ and momentum ~ 0.43 Å^{-1} each. The speed of the two-particle excitation at this point will be close to the speed u of ordinary sound. (According to the classification given in [2], this decay is of type b, and the speed of the excitation at $p \sim 0.43$ Å^{-1} differs little from u).

In the case when $Q < 0$ and the two-particle branch exists, this leads to a change of the course of the single-particle roton branch at $\varepsilon \approx 2\Delta$. The Green's function (see formula (34) of [2]) has in the weak-interaction limit the form

$$G^{-1} = \left[\omega - \epsilon_0(p) + \beta \ln\left(\frac{\alpha}{2\Delta - \epsilon}\right)\left(1 - |Q| \ln \frac{\alpha}{2\Delta - \epsilon}\right)^{-1}\right], \tag{3}$$

where $\varepsilon_0(p)$ is the "unperturbed" roton energy (i.e., without allowance for the singular graphs of Fig. 2 of [2]), and β is a positive quantity. It is seen from [3] that the single-particle branch can never intersect the two-article branch, since the equations $\gamma_2^{-1} = 0$ and $G^{-1} = 0$ have no common solutions. There are therefore two possible behaviors of the spectrum at $\varepsilon \approx 2\Delta$. Either the two-particle branch first terminates at $\varepsilon \approx 2\Delta$ with increasing momentum (at the value of the momentum at which the sign of the roton interaction Q reverses), followed by termination of the single-particle branch, or else both branches proceed almost horizontally to the point $p = 2p_0$. Since $\alpha \to 0$ as $p \to 2p_0$, the energy of the two-particle branch at this point is equal to 2Δ and this branch terminates here. The single-particle branch, on the other hand, can rise after this above the 2Δ level. One can suggest, as an interesting possibility, that this branch goes over subsequently to the spectrum of the vortex rings (see [3]).

We note, finally, one more interesting possibility that is realized if $\beta/|Q| < \Delta$ in (3). We then have a third (also single-particle) branch $\varepsilon_3(p)$ lying above the two-particle branch but above the direct branch $\varepsilon = 2\Delta$ and reaching 2Δ at the momentum value given by the equation $\varepsilon_0(p) = 2\Delta - \beta/|Q|$. All these branches are strongly undamped at $T = 0$, but have quite close energy values, and in order to observe them distinctly the temperatures may have to be quite low. A general view of the spectrum is shown schematically in the figure.

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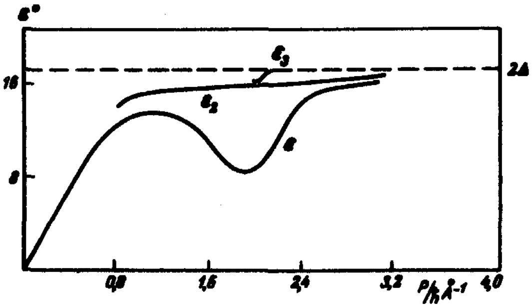
[1] Formula (1) no longer holds when the summary roton momentum p approaches zero. This, however, is of no importance in our case.

We emphasize once more that according to formula (2) the two-particle branch must lie at $\varepsilon < 2\Delta$. The experimental data of Woods and Cowley, cited in [1], offer evidence that the maximum of neutron scattering lies at $\varepsilon \sim 20°$ $> 2\Delta$. The absence of a branch with $\varepsilon < 2\Delta$ denotes in any case that the rotons are repelled. We shall show now that, regardless of the sign of Q, the cross section for the production of two rotons by a neutron with transfer of an energy $\varepsilon \geq 2\Delta$ has a maximum in this region. We note first that the cross section has a singularity at those values of the variables at which the roton-roton amplitude γ_2 has a singularity. This is seen from the diagrams of Fig. 3 of [2]. On the other hand, the γ_2 vertex has a singularity of the $\ln^{-1}[\alpha/(2\Delta - \varepsilon)]$ type at $\varepsilon \simeq 2\Delta$, as follows from (1). It can further be shown that when $\varepsilon \simeq \Delta + \Delta_m$ we get for γ_2 an expression that differs from (1) only by a factor i in front of the logarithm, so that γ_2 and the cross section have singularities of the $\ln^{-1}[\alpha'/(\Delta + \Delta_m - \varepsilon)]$ type, where $\Delta_m \simeq 13.4°$ is the roton energy at the maximum. The first singularity corresponds to a minimum of the roton production cross section, and the second corresponds either to a maximum or to a minimum. In the former case the maximum is located at $\varepsilon \simeq \Delta + \Delta_m \simeq 22°$, and in the latter case at $2\Delta < \varepsilon < \Delta + \Delta_m$. The momentum interval in which this maximum should take place is given by the inequalities



$$p_0 - p_m < p < p_0 + p_m$$

($p_m \simeq 1.1$ Å^{-1}). In addition, it is seen from formula (48) of [2] that the cross section should have a maximum on the formal continuation of the unperturbed roton spectrum beyond the 2Δ level. (Incidentally, a similar maximum should be located also on the continuation of the two-particle branch in the direction of lower momenta.) This possibly explains the results of Woods and Cowley. We note that the foregoing explanation does not contradict the assumptiontion that γ_2 has a pole in the complex plane - the undamped part of the spectrum. This assumption was advanced in [1] on the basis of an analysis of a concrete model. We state only that the theory can fix rigorously not the position of this pole, but the positions of the minima adjacent to the cross-section maximum.

In conclusion, I am grateful to I. B. Levinson for a useful discussion and to F. Iwamoto for a preprint of [1]. After this paper was sent to press, and a brief summary of the results was submitted for publication in [4], I learned of the preprint [5], in which a second branch of the spectrum, connected with roton attraction, is also considered.

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