particular, when the Fermi surface is tangent to two opposite Brillouin faces, i.e., when

$$E_F = \frac{\hbar^2 q^2}{8m} - \frac{1}{2}|W_q|$$

we get from (1)

$$\frac{\Delta g(\theta) - \Delta g(0)}{\Delta g(0)} \approx \frac{3}{8}\frac{|W_q|}{E_F}\sin^2\theta . \qquad (5)$$

Here θ is the angle between the direction of the vector $\vec{B}$ and the straight line perpendicular to the Bragg planes to which the Fermi surface is tangent. We see that the correction Δg(θ) is sharply anisotropic in this case. Therefore effects of the influence of the band structure on the g-factor should appear in experiments similar to [1] on spin resonance of the conduction electrons of a metal such as cesium. It is precisely the dependence of the g-factor on the angle θ, in accordance with formula (5), which leads to a shift of the line $\omega_s$ of the main spin resonance and of the lines corresponding to the spin waves when the magnetic field $\vec{B}$ rotates about the preferred direction in the crystal.

In conclusion, I am grateful to V.P. Silin for directing the work and to S.Z. Dunin for useful discussions.

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## LINE SHAPE OF TWO-PHOTON ABSORPTION IN A STANDING-WAVE FIELD IN A GAS

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The analysis of various nonlinear phenomena occurring when a strong field interacts with matter has been the subject of a number of investigations [1, 2]. The purpose of the present paper is to call attention to new important and physically interesting features that arise in resonant two-photon absorption in a gas in a strong standing-wave field.

1. As is well known [3], two-photon absorption is an effect of second

order in perturbation theory. Usually two-photon absorption is regarded as a process proceeding through definite real or virtual states. Assuming the attenuation of the levels to be equal to $\gamma_k$, the system of equations for the absorption probability amplitude is written in the form

$$i\hbar\left(\frac{da_k}{dt} + \gamma_k a_k\right) = \sum_m V_{km}(t)\, a_m \ . \tag{1}$$

For simplicity we confine ourselves only to three levels. We then obtain for the amplitude of the probability of two-photon absorption

$$a_3^{(2)} = \frac{e^{-\gamma_3 t}}{(i\hbar)^2} \int_0^t V_{32}(t_1)\, e^{(\gamma_3-\gamma_2)t_1} \left\{ \int_0^{t_1} V_{21}(t_2)\, e^{\gamma_2 t_2} dt_2 \right\} dt_1 \ , \tag{2}$$

where $V_{km}(t)$ is the matrix element of the Hamiltonian of the interaction of the field with the atom.

In interaction with a traveling wave, allowance for the motion of the atoms is simple, namely, the resonant frequency of the atom experiences a shift $\vec{k}\cdot\vec{v}$. As a result of averaging over all the velocities, the line shift experiences the usual Doppler broadening. We place the atomic system in a standing-wave field, which we represent as the sum of two waves traveling in opposite directions. The probability of the two-photon absorption turns out to be

$$W(2\omega) = \frac{|V_{21}|^2 |V_{32}|^2}{16\hbar^4(\omega_{21}-\omega)^2} \left\{ \frac{1}{[(\Omega-2kv)^2+\gamma_3^2]} + \frac{1}{[(\Omega+2kv)^2+\gamma_3^2]} + \frac{1}{\Omega^2+\gamma_3^2} \right\}, \tag{3}$$

where $\Omega$ is the frequency of the strong field.

The first two terms in (3) correspond to independent interaction of the atoms with two traveling waves. Each of the two traveling waves "chooses" only those atoms with which the field is at resonance with allowance for the Doppler shift $\vec{k}\cdot\vec{v}$[1]. In the case when the detuning $\Omega = \omega_{31} - 2\omega \gg \gamma_3$, the second term can be disregarded in the absorption in the field of a standing wave described by the sum of absorptions for the two traveling waves. In this case different atoms interact with each of the waves. The second term in (3) is the result of interference and plays an essential role at $\Omega \simeq \gamma_3$. The contribution of this term does not depend on the velocity of the atom. This phenomenon can be interpreted as the compensation of the Doppler shift upon interaction of the atom with the opposing waves. Being due to interference, this phenomenon arises only when it is considered in the approximation following the linear one in the field. Since the two-photon process is not observed at all in the linear approximation, it is the interference phenomenon which determine principally the line shape of the two-photon absorption in a gas in a standing-wave field.

In the c.m.s., an atom moving with velocity $\vec{v}$ is acted upon by two fields with frequencies $\omega_1 = \omega + \vec{k}\cdot\vec{v}$ and $\omega_2 = \omega - \vec{k}\cdot\vec{v}$. Therefore the sum of the frequencies $\omega_1 + \omega_2 = 2\omega$ is at resonance with the atoms, independently of their velocity, if $\omega_{31} \simeq 2\omega$. Averaging (3) over the velocities and recognizing that

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[1]An analogy can be drawn here with resonant one-photon absorption, the probability of which as a function of the velocity of the atom, as is well known [4], is described for a traveling wave by an expression that is similar to the first two terms in (3).

the power of the two-photon absorption is equal to $P(2\omega) = h\omega_{31} \quad 2\gamma_3 \quad W(2\omega)$, we obtain:

$$< P(2\omega) >_M = \frac{|V_{21}|^2 |V_{32}|^2 \omega_{31}}{8\hbar^3(\omega_{21}-\omega)^2} \left\{ \frac{\sqrt{\pi}}{2k\bar{v}} e^{-\Omega^2/(2k\bar{v})^2} + \frac{2\gamma_3}{\Omega^2 + \gamma_3^2} \right\}. \tag{4}$$

Figure 1 shows the dependence of the power of the two-photon absorption on the frequency.

2. When account is taken of the anharmonicity of the vibrations of the molecules, it is possible to regard two-photon absorption in a two-level system, assuming the dipole moments of the molecules to be oriented in a definite manner (e.g., by a constant electric field). For this case, the expression for the power of the two-photon absorption takes the form

$$< P(2\omega) >_M = \frac{|V_{21}|^2 |V_{11} - V_{22}|^2}{2\hbar^3 \omega_{21}} \left\{ \frac{\sqrt{\pi}}{2k\bar{v}} e^{-\Omega^2/(2k\bar{v})^2} + \frac{2\gamma_2}{\Omega^2 + \gamma_2^2} \right\} \tag{5}$$

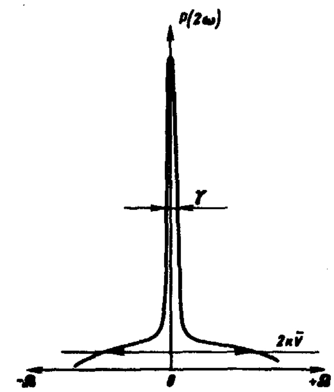


Fig. 1. Dependence of the power of two-photon absorption on the frequency. The ratio of the amplitude of the peak to the amplitude of the Doppler pedestal is $4k\bar{v}/\sqrt{\pi}\gamma$, i.e., it can be much larger than unity.

An important difference between this case and that considered above is that two-photon transitions are realized between levels on which the one-photon transition is allowed in the dipole approximation. This makes it possible to emit a photon with frequency $\omega_{12}$ in the transition 2 - 1 [5]. In the quasiclassical approximation [6] it is possible to find the dependence of the radiation power at the frequency $2\omega$ in the standing-wave field:

$$< P(2\omega) >_M = \frac{2\omega_{12}|V_{21}|^2 |V_{11} - V_{22}|^2 |P_{12}|^2}{3c^3\hbar^4\gamma_2} \left\{ \frac{\sqrt{\pi}}{2k\bar{v}} e^{-\Omega^2/(2k\bar{v})^2} + \frac{2\gamma_2}{\Omega^2 + \gamma_2^2} \right\}, \tag{6}$$

where $P_{12}$ is the matrix element of the dipole moment of the transition.

3. We calculated the power of three-photon absorption in the field of a standing wave in a gas. Its parameters differ strongly from the case of two-photon absorption. Although the probability of the absorption has a peak at the center of the line, its contrast no longer depends on the ratio $k\bar{v}/\gamma$. The physical cause of the peak at the center of the line for three-photon absorption is different than for two-photon absorption. Thus, in three-photon absorption there occurs partial cancellation of the Doppler shift, and therefore the interference term in the absorption power has a width $k\bar{v}$:

$$< P(3\omega) >_M = \frac{27|V_{21}|^2 |V_{22} - V_{11}|^2 \sqrt{\pi}}{64\hbar^5 \omega_{21}^3 k\bar{v}} \left\{ e^{-\Omega^2/(3k\bar{v})^2} + 27 e^{-\Omega^2/(k\bar{v})^2} \right\} \tag{7}$$

At the center of the line the atom interacts with both waves, so that the effective field acting on the atom increases. From this follows also a

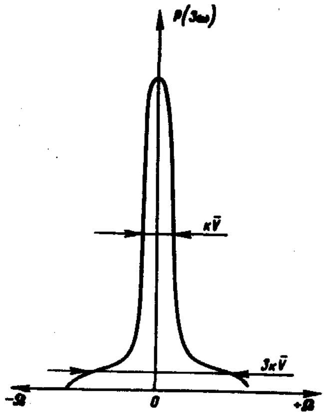

Fig. 2. Dependence of three-photon absorption power on the frequency.

decrease of the absorption at the center of the line compared with the case of an atom in the field of a traveling wave. The dependence of the three-photon absorption power on the frequency is shown in Fig. 2.

4. The foregoing effects of multiphoton absorption in gases in a standing-wave field are apparently easiest to observe by placing an absorbing medium inside a resonator. It is easy to show that a dip will appear on the dependence of the generation power on the frequency at the center of the absorption line, and that the depth of the dip can be very large.

The described phenomena are of physical interest, since they uncover in principle new possibilities for spectroscopy, frequency stabilization of powerful lasers, and a number of other fields.

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