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CHIRAL DYNAMICS OF A_1-ρ-π SYSTEM

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1. A constructive method of extracting the consequences of current algebra is the method of effective Lagrangians [1 - 5]. Our remark consists of the fact that in the usual chiral scheme for the A_1-ρ-π system [3 - 5] there is an arbitrariness connected with the presence in the Lagrangian of different terms with three and four field derivatives. In the present paper we construct an effective interaction Lagrangian of the A_1-, ρ-, and π-mesons containing not more than two derivatives in each term and giving the minimum (≤ 2) degree of the 4-momenta in all the vertices. The limitation of the dependence of the contact vertices on the 4-momenta leads to reasonable physical consequences, for example, the frequently discussed anomalous magnetic moment δ of the A_1 meson is equal to -1.

2. Assume that we have in the $SU_2 \times SU_2$ group a linearly-transforming vector in axial-vector field $\vec{\rho}_\mu$ and $\vec{a}_\mu$, and also a pion field $\vec{\pi}$ with a non-linear transformation law [1, 2]

$$\delta\vec{\pi} = -\vec{\alpha}\times\vec{\pi} + f_\pi\vec{\beta}\sigma, \quad \sigma = (1 - \vec{\pi}^2/f_\pi^2)^{1/2} \quad , \tag{1}$$

where $\vec{\alpha}$ and $\vec{\beta}$ are infinitesimal parameters and $f_\pi \simeq 95$ MeV is the pion decay constant. Then the pions enter in the Lagrangian via the covariant derivative [1 - 5]:

$$\nabla_\mu \pi^a = \{\delta_{ab} + [f_\pi^2\sigma(1+\sigma)]^{-1}\pi^a\pi^b\}(D_\mu\pi^b + g f_\pi \sigma a^b_\mu), \tag{2}$$

where a, b = 1, 2, 3 and $D_\mu\vec{\pi} = \partial_\mu\vec{\pi} - g\vec{\rho}_\mu \times \vec{\pi}$.

The invariant $(\nabla_\mu\vec{\pi})^2$ contains a non-physical bilinear coupling $\vec{a}_\mu\partial_\mu\vec{\pi}$, which is usually eliminated from L by introducing a new field

$\vec{A}_\mu = \vec{a}_\mu + (2gf_\pi)^{-1} D_\mu \vec{\pi}$ [3 - 5].

3. We call attention to the fact that after such a replacement of $\vec{a}_\mu$ from the axial-vector field self-action term $(\vec{a}_\mu \times \vec{a}_\nu)^2$ there arise in the usual Lagrangian of the A_1-ρ-π system [3 - 5] undesirable terms $[D_\mu \pi \times D_\nu \pi]^2$ with four field derivatives and terms with three derivatives. (It is convenient to introduce the symbol D for the number of derivatives in the individual terms of L, and then the usual Lagrangian [3 - 5] is characterized by the condition $D \leq 4$.) In such a case it is necessary to include in L, independently, all the possible invariants with three and four field derivatives, and then the predictions of the theory become less definite as a result of the appearance of arbitrary parameters.

Analogously, in current algebra there is a leeway in the choice of the solutions of the Ward identities [6 - 7] for the vertex functions.

It must also be stated that the presence in L of couplings with D = 3 or 4 leads to a non-smooth dependence of the $\rho \to 2\pi$, $A_1 \to 3\pi$, and $\pi\pi \to \pi\pi$ amplitudes on the 4-momenta.

In this connection we impose a natural requirement: the effective Lagrangian of the A_1-ρ-π system should contain not more than two derivatives in each term.

4. To simplify the calculations, we assume that the constants g_ρ and g_A of coupling with the vector and axial-vector fields are equal, and that the KSFR relation is satisfied [8]:

$$g_\rho = g_A = g, \qquad 2f_\pi^2 g_\rho^2 = m_\rho^2, \tag{3}$$

where m_ρ is the ρ-meson mass.

In the most general invariant Lagrangian, containing several arbitrary parameters in the terms with D = 3 or 4, these terms are eliminated in unique fashion at rigorously defined values of the parameters by means of the substitution

$$\mathbf{a}_\mu \to \mathbf{A}_\mu = \mathbf{a}_\mu + [f_\pi g(1+\sigma)]^{-1} D_\mu \pi, \tag{4}$$

in which $(2gf_\pi)^{-1} D_\mu \pi$ leads to the vanishing of the $\vec{A}_\mu D_\mu \vec{\pi}$ coupling, and the term nonlinear in the pions is determined from the requirement $D \leq 2$. It is precisely this replacement which enables us to obtain the self-action of the fields $\vec{\rho}_\mu$ and $\vec{a}_\mu$ without the appearance of terms with D = 3 or 4, with the aid of the quantities:

$$\vec{\rho}_{\mu\nu} = \partial_\mu \vec{\rho}_\nu - \partial_\nu \vec{\rho}_\mu - g\vec{\rho}_\mu \times \vec{\rho}_\nu - g(1-\sigma)\, \mathbf{A}_\mu \times \mathbf{A}_\nu + f_\pi^{-1}(\mathbf{A}_\mu \times \partial_\nu \vec{\pi} \times D_\mu \vec{\pi} \times \mathbf{A}_\nu) \tag{5}$$

$$\mathbf{A}_{\mu\nu} = \partial_\mu \mathbf{A}_\nu - \partial_\nu \mathbf{A}_\mu - g\vec{\rho}_\mu \times \mathbf{A}_\nu + g\vec{\rho}_\nu \times \mathbf{A}_\mu - g f_\pi^{-1} \vec{\pi} \times (\mathbf{A}_\mu \times \mathbf{A}_\nu) \tag{6}$$

which are chosen such that $\vec{\rho}_{\mu\nu}$ in conjunction with $\{\vec{A}_{\mu\nu} + [f_\pi(1+\sigma)]^{-1} \vec{\rho}_{\mu\nu} \times \vec{\pi}\}$ are transformed linearly in $SU_2 \times SU_2$.

The effective Lagrangian of the A_1-ρ-π system, which is quadratic in $\Delta_\mu \vec{\pi}$, $\vec{\rho}_{\mu\nu}$, and $\vec{A}_{\mu\nu}$ and contains only terms with not more than two derivatives, can be written in the form

$$L = -\frac{1}{4}\vec{\rho}_{\mu\nu}^2 - \frac{1}{4}A_{\mu\nu}^2 - \frac{1}{4}[f_\pi(1+\sigma)]^{-2}(\vec{\pi}\times\vec{\rho}_{\mu\nu})^2 - \frac{1}{2}[f_\pi(1+\sigma)]^{-1}\times$$

$$\times(\vec{\rho}_{\mu\nu}\times\vec{\pi}A_{\mu\nu}) + g(1+\sigma)^{-1}[f_\pi(\sigma-1)(D_\mu\vec{\pi}A_\mu) + (f_\pi\sigma)^{-1}(A_\mu\vec{\pi})(\vec{\pi}\partial_\mu\vec{\pi})] -$$

$$- g^2(\vec{\pi}\times A_\mu)^2 + \frac{1}{2}(1+\sigma)^{-2}[(D_\mu\vec{\pi})^2 + \frac{1}{2}f_\pi^{-2}(\vec{\pi}\partial_\mu\vec{\pi})^2] + \frac{m_\rho^2}{2}\vec{\rho}_\mu^2 +$$

$$+ \frac{M_A^2}{2}A_\mu^2 + m_\pi^2 f_\pi^2\sigma, \tag{7}$$

where m_ρ, $M_A = \sqrt{2}m_\rho$, and m_π are the masses of the corresponding particles. The choice of the term $m_\pi^2 f_\pi^2\sigma$, which violates the $SU_2 \times SU_2$ invariance, is dictated by the PCAC [1 - 2]. If we disregard the limitations (3), then the expression for the Lagrangian with $D \leq 2$ becomes somewhat more complicated and will be given in a more detailed report of the work.

5. The axial vector currents calculated from L by the Gell-Mann and Levi method satisfy the simultaneous commutation relations of the field algebra [8].

In the "tree" approximation [1 - 5], the contact vertices are polynomials in the 4-momenta, the degree of which is equal to the number of derivatives D in the corresponding term from L, and therefore the dependence of the vertices of the 4-momenta can also be characterized by the value of D. In our model, any contact vertex has $D < 2$, and for the A_1-ρ-π system this limitation cannot be made stronger, since couplings with $D = 2$ yield the terms that are needed in L, namely $(\nabla_\mu\vec{\pi})^2$, $\vec{\rho}_{\mu\nu}^2$, etc.

Thus, the vertex functions calculated from the Lagrangian (7) are a particular solution of the Ward identity in the $SU_2 \times SU_2$ field algebra with the smoothest possible dependence of the contact vertices on the 4-momenta.

6. The anomalous magnetic moment δ of the A_1 meson is not determined by the Ward identities for the $A_1A_1\rho$, $A_1\rho\pi$, and $\rho\pi\pi$ vertices [6]. Our requirement $D \leq 2$ yields in this case an acceptable value of $\delta = -1$. Without the limitations (3), it follows of necessity from the general Lagrangian with $D \leq 2$ that

$$\delta = (g_A/g_\rho)^2\{1 - [(m_\rho/f_\pi g)^2 - 1]^{-2}\} - 1 \sim -1. \tag{8}$$

We note that in this model the $\rho\pi\pi$ and $A_1 3\pi$ vertices have $D = 1$, and the contact $\pi\pi \rightarrow \pi\pi$ vertex is linear in s, t, and u. Further, for the partial widths of $A_1 \rightarrow \rho\pi$ and of the direct $A_1 \rightarrow 3\pi$ decays we obtain $\Gamma(A_1 \rightarrow \rho\pi) \simeq 45$ MeV and $\Gamma(A_1 \rightarrow 3\pi) \simeq 35$ MeV. The S-wave $\pi\pi$-scattering lengths will be the same as in the current algebra in conjunction with PCAC and the smoothness condition [8], which corresponds to our limitation on the number of derivatives in the Lagrangian. In the general Lagrangian scheme [3 - 5], at $D \leq 4$, the $\pi\pi$ scattering lengths can be arbitrarily varied by including in L the term $(\nabla_\mu\vec{\pi})^4$.

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