$$\frac{d^3\sigma(e \to e + \pi)}{ds\,dt\,dk^2} = \frac{1}{16}\left(\frac{\alpha}{2\pi}\right)^2 \left[\frac{f_\rho^2}{4\pi}\right]^{-1} \frac{1}{(s_o - M^2)^2} \frac{m_\rho^4}{k^2(k^2 + m_\rho^2)^2} \times$$

$$\times \frac{[1 + \epsilon(1-\epsilon)^{-1}]\,|a(k^2)|^2}{[s^2 - 2s(M^2 - k^2) + (M^2 + k^2)^2]^{1/2}} \frac{1}{4} \{stu - t(M^2 + k^2)(M^2 - \mu^2) - M^2(k^2 + \mu^2)^2\}. \qquad (6)$$

Here $\varepsilon(1 - \varepsilon)^{-1} = 2[(s_0 - s)(s_0 - M^2) - s_0 k^2][s^2 - 2s(M^2 - k^2) + (M^2 + k^2)^2]^{-1}$, ε is the usual degree of transverse linear polarization of the virtual photon [1], $s_0 = -(k_i + p)^2$, $k^2 = (k_i - k_f)^2$, $s + t + u = 2M^2 + \mu^2 - k^2$, and k_i and k_f are the momenta of the initial and final electrons.

The fact that the indicated processes are described by a single amplitude $a(k^2)$, and a study of reactions with linearly polarized photons yields no new information, detracts somewhat from the possibility of verifying the hypothesis of vector dominance under the assumption that the amplitude is independent of k^2. The value of the constant $f_\rho^2/4\pi$ obtained as a result of measuring the cross sections (3) and (5) can be only compared with the value extracted from data, and can be used to predict the cross section (6). On the other hand, if we forego the independence of the amplitude of k^2, then an attempt can be made to reconcile the values of the cross sections (3), (5), and (6) by approximating $a(k^2)$ by some very simple dependence, say $a(k^2) = a_0 + a_1 k^2$. Alternately, by measuring $|a(k^2)|^2$ for spatially similar k^2 and $k^2 = 0$ it is possible, given adequate statistics, to attempt to extrapolate these data to the point $k^2 = -m_\rho^2$, in analogy with the procedure used to determine the $\pi\pi$-scattering cross section. Finally, the question remains of whether $a(k^2)$ is to be regarded as a function of the variables s and t or of the variables $\nu = kp/M$ and t) besides being a function of k^2 (the changeover from s to ν introduces an additional dependence on k^2, since $s = M^2 - k^2 - 2M\nu$).

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ONE-DIMENSIONAL PROBLEM FOR S = 1 WITH MODIFIED ANTIFERROMAGNETIC HAMILTONIAN

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In [1] there was considered the Hubbard model with large Coulomb repulsion for one atom, and account was taken of the orbital angular momentum of the electron. In second order perturbation theory, this model leads to the Hamiltonian

$$H = I \sum_{r,a} P_{r,r+a} \quad . \tag{1}$$

The action of the operator $P_{r,r'}$ is as follows: $P_{r,r'}\phi_r\psi_{r'} = \phi_{r'}\psi_{r'}$ (the functions ϕ and ψ are characterized here by definite projections of the spin and of the orbital angular momentum). In particular, if L = 0, it is necessary to consider only two types of states ($S_r^z = \pm 1/2$), and then $P_{r,r'} = 2\vec{S}_r\vec{S}_{r'} + 1/2$. The antiferromagnetic character of the ground state is obvious in this case. However, if $L \neq 0$, the number of competing states is increased to six (L = 1), ten (L = 2), etc.

It is usually assumed that the orbital momenta are quenched in the crystal. However, if the crystal fields acting on the electron are small compared with the exchange energy I, then the angular momentum of the electron is weakly coupled to the lattice. We shall consider precisely this possibility.

It is quite obvious that the energy of the ground state of the Hamiltonian (1) decreases with increasing number of competing states. Thus, if the number of these states coincides with the number of nodes, then the energy of the ground state is equal to -NI. At the same time, there is a known exact result for a one-dimensional chain with L = 0 [2, 3]. The energy of the ground state is -NI(2 ln 2 - 1).

We derive in this paper equations for the energy spectrum of a one-dimensional system with three competing states. The ground-state energy of such a system is lower than the corresponding energy of an ordinary antiferromagnetic chain by an amount ∿0.3NI. This indicates that the presence of an orbital angular momentum and of weak crystal fields leads to a new type of "antiferromagnetic" state.

Let us consider a one-dimensional chain with Hamiltonian (1) and three possible single-node states. It can obviously be regarded as a chain of spins S = 1 with Hamiltonian

$$H = \sum_{j} [(\mathbf{S}_j \mathbf{S}_{j+1})^2 + (S_j S_{j+1}) - 1] \tag{2}$$

(we have put I = 1). Unlike the Heisenberg antiferromagnet, Eq. (2) contains terms $(\vec{S}_j\vec{S}_{j+1})^2$. We assume for the conditional ferromagnetic vacuum the state in which all the spin projections are equal to unity. The spin deviations can be written with the aid of the creation and annihilation operators

$$|S^z = -1\rangle_j = a_j^+ |\,bak\,\rangle_j , \tag{3}$$

$$|S^z = 0\rangle_j = b_j^+ |\,bak\,\rangle_j . \tag{4}$$

All the remaining matrix elements of the operators a and b, pertaining to one node, except those adjoint to (3) and (4), are equal to zero. The operators pertaining to different nodes commute.

We seek the eigenfunctions of the problem in the form

$$\psi = \sum f(m_1,\ldots, m_{M_1} | m_1',\ldots, m_{M_2}')\, a_{m_1}^+ \ldots a_{m_{M_1}}^+ b_{m_1'}^+ \ldots b_{m_{M_2}'}^+ |\,bak\, . \tag{5}$$

The amplitudes f are symmetrical with respect to each group indices, we can therefore assume $m_1 < m_2 < \ldots < m_{M_1}$ and $m'_1 < m'_2 < \ldots < m'_{M_2}$. In addition, we put $M_1 \leq M_2 \leq N - M_1 - M_2$, which does not limit the general character of the problem.

Following Bethe, we predefine the unphysical amplitudes

$$f(\ldots m, m \ldots | \ldots) + f(\ldots m+1, m+1 \ldots | \ldots) = 2f(\ldots m, m+1 \ldots | \ldots), \tag{6}$$

$$f(\ldots | \ldots m, m \ldots) + f(\ldots | \ldots m+1, m+1 \ldots) = 2f(\ldots | \ldots m, m+1 \ldots), \tag{6'}$$

$$f(\ldots m \ldots | \ldots m \ldots) + f(\ldots m+1 \ldots | \ldots m+1 \ldots) = f(\ldots m \ldots | \ldots m+1 \ldots) + f(\ldots m+1 \ldots | \ldots m \ldots). \tag{7}$$

The equations for the determination of the eigenvalues then simplify to

$$[E - (N - 2M_1 - 2M_2)] f(m_1, \ldots, m_{M_1} | m'_1, \ldots, m'_{M_2}) = \sum_{i,\, \alpha = \pm 1} f(\ldots m_i + \alpha \ldots), \tag{8}$$

but unphysical amplitudes appear in them, and therefore some of the eigenvalues of E corresponds to unphysical states; these, however, are readily accounted for.

We seek the solutions of (8) in the form of an expansion in plane waves

$$f(m_1, \ldots, m_{M_1} | m'_1, \ldots, m'_{M_2}) = \sum_{\{P\}} \zeta^{Q_1 \ldots Q_{M_1+M_2}}_{P_1 \ldots P_{M_1+M_2}} \exp\{ i \sum_i k_{P_i} m_{Q_i} \}. \tag{9}$$

It turns out that the problem is similar to that considered in [4] and [5]. Omitting the intermediate steps, we present the final results in the limit as $N \to \infty$, $M_1 \to \infty$, and $M_2 \to \infty$:

$$\frac{1}{1+\xi^2} = 2\pi\phi(\xi) + 4\int_{-A}^{A} \frac{\phi(\xi')\,d\xi'}{4 + (\xi - \xi')^2} - 2\int_{-B}^{B} \frac{\psi(\eta)\,d\eta}{1 + (\xi - \eta)^2}, \tag{10}$$

$$0 = 2\pi\psi(\eta) + 4\int_{-B}^{B} \frac{\psi(\eta')\,d\eta'}{4 + (\eta - \eta')^2} - 2\int_{-A}^{A} \frac{\phi(\xi)\,d\xi}{1 + (\xi - \eta)^2}, \tag{11}$$

$$\frac{M_1 + M_2}{N} = 2\int_{-A}^{A} \phi(\xi)\,d\xi, \tag{12}$$

$$\frac{M_1}{N} = 2\int_{-B}^{B} \psi(\eta)\,d\eta, \tag{13}$$

$$E = N\left(1 - 2\frac{M_1}{N} - 2\frac{M_2}{N}\right) + 4\int_{-A}^{A} \phi(\xi)\left(1 - \frac{2}{1+\xi^2}\right)d\xi. \tag{14}$$

From these equations we can obtain the energy of the ground state. It is reached at $M_1/N = 1/3$ and $M_2/N = 3$, and equals $-N(\ln 3 + (\pi/3\sqrt{3}) - 1)$.

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CP-NONINVARIANCE AND BARYON ASYMMETRY OF THE UNIVERSE

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We shall assume that at the present time the universe as a whole has a nonzero baryon number, i.e., baryon asymmetry exists, but the universe is neutral with respect to all other numbers and charges during the entire time of its existence. As to the baryon number, in the proposed model the universe is initially symmetrical with respect to it, and becomes asymmetrical only later. This is attained by introducing an interaction that does not conserve the baryon number and contains a CP-noninvariant admixture (cf. [1]). As will be shown here, the model explains in a natural manner the very occurrence of the baryon asymmetry and its magnitude.

Let us construct an interaction that does not conserve the baryon number. We write the interaction Hamiltonian in the form:

$$H = (G/\sqrt{2}) J^\mu J_\mu^+ ,$$
$$J^\mu = J_w^\mu - i\alpha T^\mu - i\alpha J_w^{\mu'} + \rho B^\mu , \qquad (1)$$
$$J_w^\mu = L^\mu + J^\mu + S^\mu, \quad J_w^{\mu'} = L^{\mu'} + J^{\mu'} + S^{\mu'} .$$

Here G is the weak-interaction constant; J_W^μ is the standard charged weak current; L^μ, J^μ, and S^μ are the lepton and strangeness-conserving and nonconserving currents, respectively; the current S^μ satisfies the rule $\Delta S = \Delta Q$. The current T^μ is the strangeness-nonconserving hadron current and satisfies the rule $\Delta S = -\Delta Q$ [2]. The current T^μ is defined in such a way that the corresponding terms in the Hamiltonian are CP-odd, and the value of the coefficient $\alpha \sim 10^{-3}$ ensures the experimentally observed magnitude of the effects of CP-nonconservation in K^0-meson decay. The additional current αJ_W^μ introduced by us is a small CP-noninvariant admixture to the standard weak current. Finally, the current B^μ does not conserve the Baryon number. Let E^μ have the following structure:

$$B^\mu = (\tilde{\kappa} C^\mu p) + (\tilde{\kappa} C^\mu \Sigma^-) + \dots \quad . \qquad (2)$$

We have introduced here a new neutral fermion κ of the Majorana type, i.e., $\kappa \equiv \tilde{\kappa}$, so that it is possible to construct states satisfying the CP equation $|\kappa\rangle = \pm|\kappa\rangle$. Thus, the current B^μ satisfies the rule $|\Delta B| = 1$, where B is the baryon number.

Thus, the Hamiltonian contains the following terms: