POSSIBILITY OF COMPLETE DRAGGING OF ELECTRONS IN A SEMIMETAL BY A LONGITUDINAL ULTRASONIC WAVE

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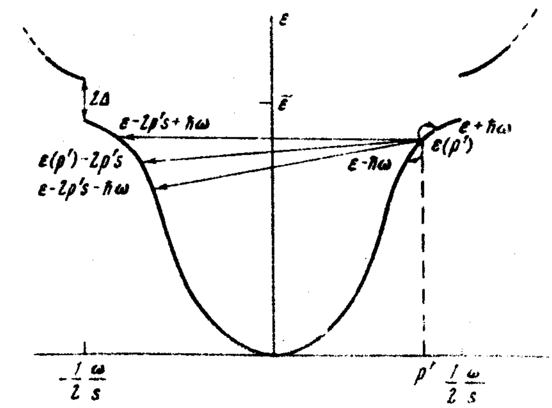
In the analysis of problems connected with the propagation of ultrasound in a metal it is customary to regard sound that changes the electron energy by an amount $\Delta = \gamma\varepsilon_0$ (γ - elastic deformation, $\varepsilon_0 \sim 1 - 10$ eV) as a small perturbation, and the electron density matrix, for example, is expanded in powers of Δ [1]. In some cases it is more advantageous to use a different formulation of the problem: find first the energy levels of the electron in the crystal deformed by the sound, and then take into account the influence of collisions with impurities (with frequency ν_n) or electron-electron collisions (with frequency ν_{ee}). The limited applicability of the first approach is due to the fact that the choice of the equilibrium value of the density matrix in the unperturbed crystal as the zeroth approximation presupposes implicitly that the energy and momentum transferred from the sound wave to the individual electrons relax sufficiently rapidly in the electron system. But the frequencies ν_n and ν_{ee}, which are responsible for the relaxation, can be small, and



Energy spectrum of electrons in the "sound frame." The arrows show the transitions in the case of scattering by impurities. $\tilde{\varepsilon} = (1/2)m(\hbar\omega/2s)^2$.

therefore a certain group of carriers (we are interested in particles having a wavelength equal to double the sound wavelength) can be dragged by a sufficiently intense sound wave (for which, of course, the condition $\gamma \ll 1$ is still satisfied).

In the second formulation of the problem, the cases of standing and traveling sound waves differ significantly. For an electron in the field of a traveling longitudinal wave, the Schrodinger equation, in the simplest case of a quadratic dispersion law, is

$$i\hbar \frac{\partial \psi}{\partial t} = \left\{ \frac{p^2}{2m} + \Delta \cos\left[\frac{\omega}{s}(x - st) \right] \right\} \psi, \qquad (1)$$

where ω and s are the frequency and speed of the sound, respectively. This equation was used by Keldysh [2] to calculate the dragging of electrons by a sound wave in a semiconductor. We shall consider the case of a semimetal at low temperatures ($T \to 0$). It is apparently possible to realize conditions wherein the electrons in a semimetal are dragged by the wave completely, i.e., the sound wave produces a direct current with approximate density nes (n - number of carriers per unit volume, e - electron charge). We are referring to a semimetal placed in a strong magnetic field ($\sim 10^5$ Oe), when the electron gas becomes "one-dimensional" and the Fermi surfaces (FS) cylindrical: one lower Landau level is occupied, and the electron energy depends only on the projection of the momentum on the direction of the magnetic field (cf., e.g., [3]). The magnetic field must be oriented along the sound propagation direction.

To solve Eq. (1) (which is now one-dimensional), it is convenient to change over, as in [2], to the "sound frame," i.e., make the change of variable $y = x - st$. It suffices to solve the transformed equation by means of ordinary perturbation theory for degenerate levels [4]. The spectrum has a gap whose position is determined by the frequency of the sound (see the figure). Alignment of the gap with the FS leads, as we shall show, to a maximum dragging of the electrons.

We now find the electron density matrix from the Liouville equation (written in the interaction representation)

$$i\hbar \frac{\partial \rho}{\partial t} = [V, \rho] \qquad (2)$$

V is the potential of the interaction with the impurities (when $T \to 0$, the impurities play the main role). We use as the model randomly distributed δ-like potentials. We transform (2) into an integral equation and average over the impurity distribution:

$$\rho(t) = \rho(t_0) - \hbar^{-2} \int_{t_0}^{t} dt_1 \int_{t_0}^{t} dt_2 \, [\overline{V(t_1), [V(t_2)}, \rho(t_2)]].$$

To determine the form of the function $\rho(t)$ in the steady state, it suffices to find the asymptotic form as $t \to \infty$. For the case

$$\Delta >> \hbar\omega >> \hbar\nu_n \qquad (3)$$

and the result takes the form

$$\rho_{mn} = \delta_{mn}\rho(\epsilon_n),\ \rho_\pm(\epsilon) = C_1 + C_2\left\{f(\epsilon) \pm \frac{\hbar\omega}{2}\frac{[(\tilde{\epsilon}-\epsilon)^2 - \Delta^2]^{1/2}}{\epsilon} f'(\epsilon)\right\},$$
$$f(\epsilon) = \exp\left\{-\frac{2}{3}\frac{(\tilde{\epsilon}-\epsilon)^3}{\Delta^2\epsilon_F}\right\},\ \epsilon \lesssim \tilde{\epsilon} - \Delta\ . \qquad (4)$$

The ± signs correspond to the two branches of the function $p(\varepsilon)$ in the figure, and ε_F is the Fermi energy.

The solution does not coincide with the Fermi function, because the scheme of transitions between the levels in the case of scattering by impurities in the sound frame (see the figure) is more complicated than for the case of free electrons. The result (4) was obtained neglecting the electron-electron collisions (for $\nu_{ee} << (\hbar\omega/\Delta)^2\nu_n$). The constants C_1 and C_2 in (4) at T = 0 are determined from the condition $\rho(\varepsilon) \to 1$ as $\varepsilon \to -\infty$ and from the condition for the normalization of ρ.

It can be seen from (4) that direct current is produced (which is nondissipative if ν_{ee} is neglected) in the reference frame in which the crystal is at rest. The maximum current is j_{max} = nes and occurs when $(\omega/2s)$ coincides with p_F (the Fermi momentum in the absence of sound, the gap falls on the FS), and decreases linearly with increasing sound frequency:

$$j \sim j_{max}\left[1 - (\epsilon_F/\Delta)^{2/3}\frac{k - p_F}{p_F}\right].$$

Numerical estimates show that the conditions described above can be realized experimentally. The requirement $(\omega/s)^{-2} \sim p_F$ corresponds to $\omega \sim 10^{11}$ sec^{-1} in the case of bismuth. The conditions (3) stipulate $\nu_n << 10^{11}$ sec^{-1} and $\Delta >> 10^{-16}$ erg. The equivalent inequality for the intensity of the sound flux S is $S >> 2$ W/cm². To observe a large dragging current it is necessary to have $T \lesssim \Delta$, which yields at $T \sim 1°K$ approximately the same estimate.

If there is no magnetic field, the dragging current will apparently exist[1)] but be smaller in magnitude, $\sim nes(\Delta/\varepsilon_F)$.

Direct current can flow in the sample, of course, only if the electric circuit is closed. In an open sample, a constant electric field $E \sim j/\sigma$ is produced (σ is the conductivity).

In conclusion, let us discuss briefly the case of a standing sound wave. Since $s/v << 1$ (v - electron velocity), we can speak of an "adiabatic" electron spectrum with a gap whose magnitude varies at the sound frequency. The existence of the gap should become manifest in experiment, for example, by the appearance of a dependence of the period of the de Haas - van Alphen oscillations on the sound frequency.

[1)]We do not discuss here the question of the influence of many-particle interactions on the gap in the spectrum.

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