GIANT QUANTUM OSCILLATIONS OF SOUND ABSORPTION BY A METAL FILM IN A MAGNETIC FIELD

V.A. Margulis

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When sound is absorbed by a metal in a quantizing magnetic field, giant oscillations of the sound-absorption coefficient are produced [1]. As shown in [1], these oscillations should depend on the angle between the directions of the field and of the sound wave. No such oscillations are produced if this angle equals 90°.

Allowance for the influence of the boundaries on the sound propagation may change the situation, and oscillations can arise also in the case when the field and sound-wave directions are mutually perpendicular, owing to the non-conservation (as a result of the uncertainty principle) of the z-projection of the quasimomentum in the scattering of a sound quantum by an electron [2] ($0 \leq z \leq L_z$).

In this paper we consider the absorption of sound by a free film in the case when the magnetic field and the sound-propagation velocity lie in the plane of the film and their directions are mutually perpendicular. It is shown that, owing to the change of the energy spectrum of the electron gas (lifting of the degeneracy with respect to one quantum number because of the influence of the boundaries), the absorption coefficient experiences the oscillations of Gurevich, Skobov, and Firsov, in spite of the fact that, unlike in [2], the quasimomentum is conserved in the direction where the oscillations occur.

The wave functions and the energy spectrum of the electron gas for our case were found in [3] in the case of a parabolic well. For the case when the magnetic field is directed along the x axis, we can write

$$\psi_{Mp_xp_y} = \frac{1}{\sqrt{L_xL_y}} \exp\left\{\frac{i}{\hbar}(p_x x + p_y y)\right\} \frac{1}{\sqrt{\ell}}\phi_M\left(\frac{z - z^0}{\ell}\right),$$

$$\epsilon_{Mp_xp_y} = \hbar\tilde{\omega}\left(M + \frac{1}{2}\right) + \frac{p_x^2}{2m^*} + \frac{\omega_0^2}{\tilde{\omega}^2}\frac{p_y^2}{2m^*}, \qquad (1)$$

where M is the magnetic-film number $\omega_c = eH/m^*c$ the magnetic-film frequency is $\tilde{\omega} = (\omega_0^2 + \omega_c^2)^{1/2}$, $z_0 = (c/eH)(\omega_c/\tilde{\omega})^2 p_y$, ϕ_M is the oscillator function, and the magnetic-film length is $\ell = (\hbar/m^*\omega)^{1/2}$. For simplicity we consider the case $m = m^*$. In order for the model to describe well the lower level, it is necessary to use for the estimates $\omega_0 \sim (\hbar/m^*)(\pi/L_z)^2$, and for the upper ones $\omega_0 \sim (2\xi/m^* L_z^2)^{1/2}$, where ξ is the Fermi energy. For the case of interest to us, that of a thick film with free boundaries, we have $\omega_c >> \omega_0$.

We shall assume that the wave is directed along the y axis, and then the laws for the conservation of the energy and the longitudinal quasimomentum yield

$$\hbar\tilde{\omega}(M + 1/2) + \frac{p_x^2}{2m^*} + \left(\frac{\omega_0}{\tilde{\omega}}\right)^2 \frac{p_y^2}{2m^*} + \hbar\omega = \hbar\tilde{\omega}(M' + 1/2) + \frac{p_x^2}{2m^*} + \frac{\omega_0^2}{\tilde{\omega}^2} \frac{(p_y + \hbar\kappa)^2}{2m^*} \left(\frac{\omega_0}{\omega}\right)^2, \tag{2}$$

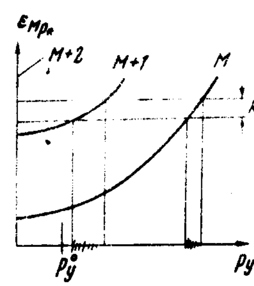
assuming that

$$\epsilon_{Mp_x}(p_y + \hbar\kappa) = \epsilon_{Mp_x}(p_y) + \frac{\hbar p_y \kappa}{m^*}\left(\frac{\omega_0}{\tilde{\omega}}\right)^2$$

we find that if the magnetic field is so strong that

$$\tilde{\omega} > \frac{\kappa p_F}{m^*}\left(\frac{\omega_0}{\tilde{\omega}}\right)^2, \text{(i.e. } M = M'\text{)},$$
$$p_y = p_y^0 = \frac{m^* w_p}{\omega_0^2}\tilde{\omega}^2, \tag{3}$$

where $\omega = w_p \kappa$ is the sound frequency and κ is the wave vector. Consequently, only electrons with $p_y = p_y^0$ can take part in the absorption. If $L_z \to \infty$, corresponding to a changeover to a bulky sample, then p_y^0 becomes much larger than p_F, and therefore the electrons cannot absorb the sound. For absorption it is necessary that p_y^0 fall in the hatched intervals of the values of p_y in the figure.



Let us write down the expressions for the sound absorption coefficient [1]

$$\Gamma = \frac{\pi}{V\rho u_0^2 \omega} \sum_{Mp_xp_y} \sum_{M'p_x'p_y'} \frac{\partial F(\xi - \epsilon_{Mp_xp_y})}{\partial \xi} \left|\langle Mp_xp_y | u | M'p_x'p_y' \rangle\right|^2 \times \delta[\hbar\tilde{\omega} + (\epsilon_{Mp_xp_y} - \epsilon_{M'p_xp_y})], \tag{4}$$

where V is the volume, u_0 the amplitude of the oscillations in the sound wave, w the group velocity of the sound, F the Fermi function, and ρ the density of the medium. The square of the matrix element of scattering in [4] can be easily found in similar fashion [1].

If we take a Fermi function in the form of a step, then we can get from (4)

$$\Gamma = \Gamma_0 \left(\frac{\tilde{\omega}^2}{\omega_0 \omega_c}\right)^2 \frac{1}{2L_z} \sum_M \int dp_x \, \delta \left[\xi - \hbar\tilde{\omega}(M + 1/2) - \frac{p_x^2}{2m^*} - \right.$$

$$\left. - \frac{\tilde{\omega}^2}{\omega_0^2} \frac{m^*}{2\hbar^2 \kappa^2} \left(\frac{\hbar^2 \kappa^2}{2m^*} - \hbar\omega\right)^2 \right], \tag{5}$$

where Γ_0 is the absorption coefficient in the unquantized spectrum [4]. If we neglect in (5) the last term of the δ function, which is small compared with ξ (this corresponds to a small shift of the oscillation picture), and also recognize that $\omega_c >> \omega_0$, then we obtain

$$\Gamma = \Gamma_0 \left(\frac{\omega_c}{\omega_0}\right)^2 \frac{1}{L_z} \sum_M \sqrt{\frac{m}{2[\xi - \hbar\omega(M + 1/2)]}}, \tag{6}$$

It is seen from (6) that oscillations of Γ take place with a period $\Delta(1/H) = e\hbar/m^*c\xi$. To estimate Γ^{max} it is necessary to take into account in (4) the thermal development of the distribution function F, and then

$$\Gamma = \Gamma_0 \left(\frac{\omega_c}{\omega_0}\right)^2 \frac{\kappa}{8L_z m^{*2} kT} \sum_M \int dp_x dp_y \, \delta \left[\frac{\kappa p_y}{m^*} + \left(\frac{\tilde{\omega}}{\omega_0}\right)^2 \times \right.$$

$$\left. \times \left(\frac{\hbar\kappa^2}{2m^*} - \tilde{\omega}\right) \right] \mathrm{ch}^{-2} \left\{ \frac{1}{2kT} \left[\xi - \hbar\tilde{\omega}(M + 1/2) - \frac{p_x^2}{2m^*} \right] \right\}, \tag{7}$$

where we have neglected in the argument of the hyperbolic cosine the small quantity

$$\left(\frac{\omega_0}{\tilde{\omega}}\right)^2 \frac{p_y^2}{2m^*},$$

which causes a small shift of the oscillation peaks. If, however, we take this quantity into account, then as $L_z \to \infty$ we can readily see that $\Gamma \to 0$, corresponding to the transition to the bulky sample.

In addition, to obtain Γ^{min}, it is necessary to take into account in (5) the corrections connected with the scattering, i.e., replace the δ function by

$$\frac{1}{\pi} \frac{1/\tau}{\left[\frac{\hbar\kappa p_y}{m^*} + \left(\frac{\tilde{\omega}}{\omega_0}\right)^2 \left(\frac{\hbar\kappa^2}{2m^*} - \omega\right) \right]^2 + \frac{\pi\hbar}{\tau^2}}. \tag{8}$$

Using even a scheme analogous to (1), we find that $\Gamma^{max}/\Gamma^{min} >> 1$.

[1] V.L. Gurevich, V.G. Skobov, and Yu.A. Firsov, Zh. Eksp. Teor. Fiz. 40, 786 (1961) [Sov. Phys.-JETP 13, 552 (1962)].
[2] B.A. Tavger and V.A. Margulis, ibid. 58, 635 (1970) [31, 340 (1970)].
[3] E.M. Lifshitz and A.M. Kosevich, Dokl. Akad. Nauk SSSR 91, 705 (1953).
[4] A.I. Akhiezer, M.I. Kaganov, and G.Ya. Lyubarskii, Zh. Eksp. Teor. Fiz. 32, 837 (1957) [Sov. Phys.-JETP 5, 685 (1957)].

ERRATUM

In the article by V. A. Margulis, Vol. 12, No. 5, p. 186, the factor h/m* was omitted from formulas (5) and (6). In addition, the factor h was omitted from formula (4) (p. 185) and formula (7). Formula (8) should read

$$\frac{1}{\pi}\,\frac{1/r}{\left[\dfrac{\kappa P_y}{m^*}+\left(\dfrac{\tilde{\omega}^2}{\omega_0^2}\right)^2\left(\dfrac{\hbar k^2}{2m^*}-\tilde{\omega}\right)\right]^2+\dfrac{1}{r^2}}\;.$$

NOTE

For technical reasons, the balance of the Russian version of Volume 12, Number 11 will be published in Volume 12, Number 12 of the translation.