Let us consider the tunnel current in an SIM junction at temperatures $T > T_c$ (T_c is the critical temperature of the metal M in this junction). Using the expression for the density of state of the electrons in the superconductor (cf., e.g., [5]), we can write the tunnel current in the form

$$jR_o = -\frac{1}{2}\int_{-\infty}^{\infty}\left(\text{th}\frac{\epsilon - V}{2T} - \text{th}\frac{\epsilon}{2T}\right)\frac{|\epsilon|\,\theta(|\epsilon| - \Delta)}{(\epsilon^2 - \Delta^2)^{1/2}} \times$$

$$\times\left\{1 + \frac{4\sqrt{2}T^2}{mp_o P^{3/2}}\frac{1}{(\epsilon - V)^2}\left[\Gamma^{1/2} - \frac{[((\epsilon - V)^2 + \Gamma^2)^{1/2} + \Gamma]^{3/2}}{2\sqrt{2}((\epsilon - V)^2 + \Gamma^2)^{1/2}}\right]\right\} \qquad (7)$$

This integral can be calculated only in particular cases, making a detailed comparison of (7) with experiment [7] difficult. It is readily seen, however, that for an applied potential $V < \Delta$ the fluctuation correction to the current is negligibly small. A fluctuation current begins to appear when V becomes equal to Δ ($\Delta - V \sim T - T_c$); this is in qualitative agreement with experiment [7]. The fluctuation correction to the tunnel current at $V = \Delta$ is proportional to $T/(T - T_c)$.

In conclusion, it is my pleasant duty to thank Yu.N. Ovchinnikov, D.E. Khmel'nitskii, and G.M. Eliashberg for a discussion of the results.

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CONTRIBUTION TO THE NONLINEAR THEORY OF EXCITATION OF A MONOCHROMATIC PLASMA WAVE BY AN ELECTRON BEAM

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It is well known that when a monochromatic wave interacts with a plasma the capture of the resonant plasma particles in the potential well produced by the wave leads to oscillations of the wave amplitude with a characteristic time on the order of the period of the particle oscillations in the well, $\tau_0 = k^{-1}(e\phi_0/m)^{-1}$ (k - wave number, ϕ_0 - amplitude of potential in the wave). The oscillations are attenuated as a result of the phase "mixing" of the captured particles, due to the dependence of the period of their oscillations in the well on the energy. These features of the interaction of the monochromatic wave with a plasma were explained by Mazitov [1] and O'Neil [2], who investigated the absorption of a wave with a sufficiently small decrement $\gamma_L\tau_0 << 1$. In this case it was possible to obtain an approximate analytic solution of the problem by considering the motion of resonant particles in a field of given amplitude, and then taking into account the effect of their reaction on the wave.

In this paper we consider the nonlinear theory of excitation of a monochromatic plasma wave by means of a beam in two limiting cases: when the electron beam is "smeared," $\Delta v/v_0 >> (n_1/n_0)^{1/3}$, and "monoenergetic," $\Delta v/v_0 << (n_1/n_0)^{1/3}$ (Δv is the velocity spread in the beam, v_0 the mean velocity, and n_1 and n_0 the beam and plasma densities, with $n_1 << n_0$)[1]. In a low-density beam, the amplitude of the excited wave is also sufficiently small ($e\phi_0 << mv_0^2$), and the linear approximation is valid for the thermal particles that determine the dispersion of the oscillations. Only the motion of the resonant particles with velocities $|v - v_{ph}| \lesssim (5\gamma_L/k)$, which cause the buildup of the wave amplitude, is essentially nonlinear ($v_{ph} = \omega_0/k$ is the phase velocity of the wave).

In the case of a "smeared" beam, when the instability of the beam in the plasma is kinetic ($\gamma_L = (2\pi^2e^2/mk)\omega_0(\partial f/\partial v_{ph})$, where f_0 if the equilibrium distribution function of the resonant particles), the system of equations describing the excitation of the monochromatic wave is

$$\frac{dv}{dt} = -\frac{e}{m}E(t)\sin kz, \qquad \frac{dz}{dt} = v, \tag{1}$$

$$\frac{1}{4\pi}E\frac{dE}{dt} = -\overline{j^{res}E(t, z)} = \frac{4\pi e}{\lambda}E(t)\int_{-\lambda/2}^{\lambda/2} dz_0 \int_{-v_0^m}^{v_0^m} dv_0 \sin kz(v + v_{ph}) \times f_0(v + v_{ph}). \tag{2}$$

Equations (1) and (2) are written in the reference frame of the wave, j^{res} is the current of resonant particles, the bar corresponds to averaging over the wavelength γ, and z_0 and v_0 are the initial values of the coordinate and velocity of the particle located at the point (z, v) of phase space at the instant t. In deriving (2) we used the Liouville theorem concerning the conservation of phase volume, $dzdv = dz_0dv_0$, and the condition for the conservation of the distribution function on the particle trajectory, $f(t, z, v + v_{ph}) = f_0(v_0 + v_{ph})$ (the initial perturbation of the equilibrium distribution function in (2) can be neglected).

At small amplitudes, when the width of the region where the particles are captured by the wave is much smaller than the interval of the resonant particle velocities, $\sqrt{e\phi_0/m} < \gamma_L/k$, the system (1) - (2) describes the exponential growth of the amplitude during the instability. The amplitudes attained in the course of time,

$$\phi_0 \sim \frac{m\gamma_L^2}{ek^2}, \tag{3}$$

[1]During the course of this work, the authors learned that the nonlinear theory of excitation of monochromatic waves in a plasma is being considered also by R.Z. Sagdeeva and by B. Fried.

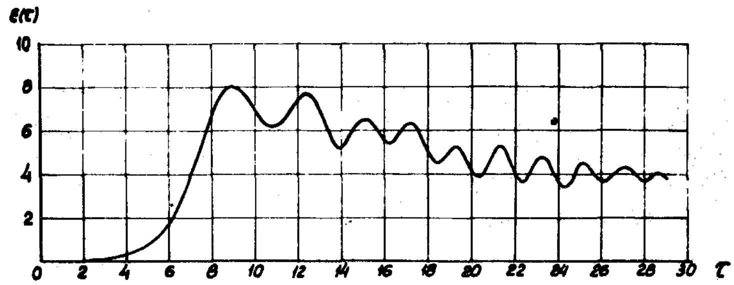

Fig. 1

are such that an appreciable part of the resonant particles is captured in the potential well produced by the wave. The oscillations then stop growing, and the captured particles give rise to amplitude oscillations that attenuate as a result of the phase "mixing" of these particles. An estimate of the maximum oscillation amplitude, which coincides with (3), was presented earlier in [3, 4].

Since there is no small parameter in the problem of oscillation excitation ($\gamma_L\tau_0 \gtrsim 1$), the solution of the system (1) - (2) can be obtained only by numerical methods. In terms of the dimensionless variables

$$\nu = \frac{1}{2\pi}\frac{kv}{\gamma_L}, \quad \zeta = \frac{1}{2\pi}kz, \quad \tau = \gamma_L t, \quad \epsilon = \frac{eEk}{m\gamma_L^2}$$

we can write the system (1) - (2) in the form

$$\frac{d\nu}{d\tau} = -\frac{\epsilon}{2\pi}\sin 2\pi\zeta, \qquad \frac{d\zeta}{d\tau} = \nu, \tag{4}$$

$$\frac{d\epsilon}{d\tau} = 16\pi\int_0^{1/2} d\zeta_0 \int_{-\nu_0^m}^{\nu_0^m} d\nu_0\, \nu_0 \sin 2\pi\zeta, \tag{5}$$

where we use the condition $\Delta v \gg (5\gamma_L/k)$, represent the resonant-particle distribution function in the form

$$f_0(v + v_{ph}) = f_0(v_{ph}) + v\frac{\partial f_0}{\partial v_{ph}}$$

and eliminate in (5) the integral over $\zeta < 0$ with the aid of the condition

$$\nu(-\zeta_0, -\nu_0, \tau) = -\nu(\zeta_0, \nu_0, \tau), \quad \zeta(-\zeta_0, -\nu_0, \tau) = -\zeta(\zeta_0, \nu_0, \tau).$$

The system (4) - (5) was integrated with a computer. We integrated the trajectories of 1528 particles with initial velocities in the range $-2 \le \nu_0 \le 2$ in steps $\Delta\nu_0 = 2/95$, and with initial coordinates in the range $0 \le \zeta_0 \le 1/2$ in steps $\Delta\zeta_0 = 1/14$. Figure 1 shows the obtained plot of $\varepsilon(\tau)$ at $\varepsilon(0) = 10^{-2}$ and an optimal spacing $\Delta\tau = 5 \times 10^{-3}$. At $\tau \lesssim 7$, this plot has an exponential character, $\varepsilon \sim \exp(0.89\,\tau)$ (at the chosen value of the maximum resonant-particle velocity, the linear growth increment is $\gamma = 0.90\gamma_L$). This growth subsequently slows down, the amplitude of the oscillations reaches the first maximum

$\varepsilon^{(1)}_{max} = 8.2$ at $\tau = 8.8$, and oscillations of the amplitude set in at large τ. The decrease of the mean value of ε, about which these oscillations take place, is possibly due to the onset of instability due to acceleration of the captured particles in the oscillating-amplitude field. With increasing τ, the amplitude of the oscillations decreases, and when $\tau > 25$ it is possible to regard ε as approximately constant, $\varepsilon_{stat} \approx 4$, corresponding to an oscillation energy

$$\frac{E^2}{4\pi} = \epsilon^2_{stat} n_1 m v \Delta v \left(\frac{\gamma_L}{k \Delta v}\right)^4 \quad (6)$$

much lower than the beam energy.

In the case of a monoenergetic beam, the instability leads primarily to excitation of a wave with $k = \omega_0/v_0$, corresponding to a maximum of the linear increment $\gamma = 0.686\omega_0(n_1/n_0)^{1/3}$. The frequency of the wave is close to the resonant frequency of the plasma and the excitation of the higher harmonics of frequency $\omega = n\omega_0$ with $n \geq 2$ can be neglected (their amplitude is $E_n \sim E_1(\gamma/\omega)$). By representing the electric field of the wave in the form

$$E(t, z) = E(t) \sin[k(z - vt) + \alpha(t)] \quad (7)$$

and recognizing that in the case of a "monoenergetic" beam the time variation of not only the field amplitude E but also of its phase α is determined by the interaction of the beam particles with the wave, we obtain the following system of equations for the amplitude and the phase

$$\frac{du}{d\tau} = -\epsilon \sin(2\pi\zeta + \alpha); \qquad \frac{d\zeta}{d\tau} = \frac{1}{2\pi} u, \quad (8)$$

$$\frac{d\epsilon}{d\tau} = \int_{-1/2}^{1/2} \sin(2\pi\zeta + \alpha)\, d\zeta_0; \quad (9)$$

$$\left(\frac{d\alpha}{d\tau} - \delta\right)\epsilon = \int_{-1/2}^{1/2} \cos(2\pi\zeta + \alpha)\, d\zeta_0 . \quad (10)$$

In these equations we used the dimensionless variables

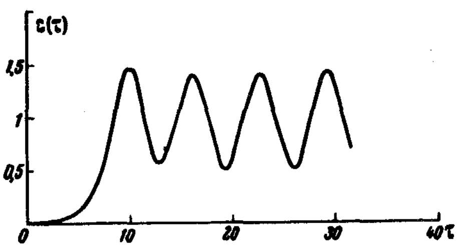

Fig. 2

$$\tau = \omega_0\left(\frac{n_1}{n_0}\right)^{1/3} t; \quad \delta = \frac{\omega_0 - k v_0}{\omega_0\left(\frac{n_1}{n_0}\right)^{1/3}}; \quad \zeta = \frac{z - v_0 t}{\lambda}; \quad u = \frac{v - v_0}{v_0\left(\frac{n_1}{n_0}\right)^{1/3}};$$

$$\epsilon = \frac{E}{\sqrt{4\pi n_1 m v_0^2 \left(\frac{n_1}{n_0}\right)^{1/3}}}$$

and in their deviation it was assumed that at $t = 0$ the beam has a "δ-like" velocity distribution $f_0(t = 0) = n_1\delta(v - v_0)$. The system (8) - (10) was integrated by the Runge-Kutta method with $\delta = 0$ and $\varepsilon_0 = 10^{-2}$. We integrated the trajectories of 100 particles with initial $-1/2 \leq \zeta_0 \leq 1/2$.

The results of this integration are shown in Fig. 2. At $\varepsilon << 1$ the amplitude has an exponential growth in accord with the linear theory ($\varepsilon \sim \exp(0.683\tau)$). When $\varepsilon \sim 1$, amplitude oscillations are produced by the capture of the beam in the potential well of the wave. In this case there is no noticeable damping of the oscillations as a result of phase "mixing" of the captured particles. The energy of the wave excited by the monoenergetic beam is quite appreciable

$$\frac{E^2}{4\pi} = \varepsilon^2 n_1 m v_0^2 \left(\frac{n_1}{n_0}\right)^{1/3} \qquad (11)$$

where ε ranges from 1.5 to 0.5.

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ENERGY SPECTRA OF PARTICLES OBSERVED IN REACTIONS PROCEEDING VIA A COMPOUND NUCLEUS

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The energy distribution of the nucleons emitted by a compound nucleus is satisfactorily described by the Maxwellian distribution $\sigma(E_f) \sim \exp(-E_f/T)$, with a temperature $T \sim 0.5 - 2.0$ MeV [1]. The only exception is the hard part of the spectrum with energy $E_f \lesssim E_i$ (E_i is the energy of the incident particle), where $\sigma(E_f)$ is a nonmonotonic function [2]. We consider here an effect that makes it possible to explain qualitatively the difference between the observed spectrum and the evaporation spectrum.

It is easy to verify that a compound nucleus is a normal Fermi system, since the temperature of the phase transition from the superfluid state into the normal state, $T_e \simeq 0.57\Delta$ (2Δ is the energy gap and is of the order of 0.5 - 1.5 MeV in even-even nuclei), is as a rule smaller than T. If the particle leaves the nucleus with a velocity $v >> R\Delta$ (i.e., within times much shorter than the pairing relaxation time $\sim 1/\Delta$), then the ensuing cooling of the nucleus can be regarded as instantaneous. The normal state $|i\rangle$ that is produced thereby is metastable with a width $\Gamma = \Delta$, and represents a superposition of the ground and excited states of the superfluid nucleus $|f\rangle$[1)].

1) We note that the competing process, i.e., the emission of a nucleon from a superfluid nucleus, is less probable in our case, since the normal state lies higher in energy: $\delta E = E_N - E_S = (\rho\Delta)(\Delta/2) > 0$ (ρ is the density of the single-particle levels at the Fermi boundary) and $\exp[-(E_f - E_N)/T] > \exp[-(E_f - E_S)/T]$.