$$f - 2\xi f' = (f^{5/2} f')' - k^2 f^{-3/2}(f - g) ,$$
$$g - 2\xi g' = Z k^2 f^{-3/2}(f - g) , \tag{2}$$
$$\frac{3}{2} \int_{-\infty}^{\xi_0} (Zf + g) d\xi = 1,$$

where ξ_0 is the dimensionless coordinate of the front of the thermal wave.

The approximate solution of the system (2) yields $\xi_0 \simeq 0.66Z^{-5/9}$ and an average dimensionless ion "temperature" $\bar{g} \simeq 0.5k^2 Z^{8/9}$. Turning to (1), we can readily see that the dimensional ion temperature in the thermal-wave regime decreases with increasing energy absorption[1)], $T_i \sim Q^{-2/9}$. Since T_i has a maximum as a function of Q (for a given pulse duration). An estimate shows that the maximum of T_i is attained at $k^2 \simeq 1.4Z^{-4/3}$, corresponding to a case intermediate between (i) and (ii). The ion temperature is in this case approximately one-third the electron temperature.

It is of interest to note that the competition between the hydrodynamics and the heat conduction is determined by the same parameter as the ratio of the electron and ion temperatures. The thermal-wave mode corresponds to appreciable superheating of the electrons relative to the ions. The situation is qualitatively the same in the case of ultrashort pulses, where the hydrodynamic motion becomes noticeable after equalization of the electron and ion temperatures [2].

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MASS OPERATOR AND EXACT GREEN'S FUNCTION OF AN ELECTRON IN AN INTENSE FIELD

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As is well known, the radiative corrections to the motion of an electron in an external field can be described by the mass operator in the modified Dirac equation [1] and can be experimentally detected by the effects of scattering and level shifting. The mass operator has been considered in detail only for a Coulomb field in the lower orders in this field and in the radiation field.

Although the presently attained electromagnetic fields F are weak compared with the characteristic quantum-electrodynamics value $F_0 = m^2c^3/eh = 4.4 \times 10^{13}$ Oe, it is possible to obtain a field of the order of F_0 in the rest system of a high-energy electron moving in a field of high intensity. Regardless of the

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1) We refer here, of course, to the temperature reached during the time of action of the light pulse. After the end of the pulse, the process evolves qualitatively in the same manner as for an ultrashort pulse.

form of the field in the laboratory system, in the electron's own system it will be very close to the field of a plane wave, and if its characteristic wavelength and period are large compared with m/eF, then it can be regarded as a constant crossed field ($\vec{F} \perp \vec{H}$, E = H). We describe in this paper the structure and the properties of the mass operator, and of the eigenfunctions and Green's functions of an electron in a crossed field; these functions are exact in terms of the external and radiation fields.

If we write down the Volkov solution of the Dirac equation in a crossed field in the form $\psi_p(x) = E_p(x)u_p$, cf. [2], and continue analytically the matrix $E_p(x)$ into the region $p^2 \neq -m^2$, then a Fourier transformation in which $E_p(x)$ is used in lieu of the usual e^{ipx} makes the mass operator diagonal in p_μ and transforms the modified Dirac equation into[1)]

$$[i\gamma p + m + M(p, F)]\phi(p) = D(p, F)\phi(p) = 0.$$

In a crossed field we have

$$D = S + i\gamma V + \sigma T + i\gamma_5\gamma A, \quad S = ms, \quad V_\mu = p_\mu v + (e^2F_{\mu\nu}F_{\nu\lambda}p_\lambda/m^4)v_2,$$

$$T_{\mu\nu} = (eF_{\mu\nu}/m)t, \quad A_\mu = (eF^*_{\mu\nu}p_\nu/m^2)a;$$

the lower-case letters denote scalar functions of p^2 and $\chi = \pm\sqrt{(eF_{\mu\nu}p_\nu)^2/m^3}$. The solution of the modified equation exists only if

$$\det D = (S^2+V^2+A^2)^2 - 4(SA - 2T^*V)^2 = 0, \text{ i.e.,} \tag{1}$$

$$m^2s^2 + p^2v^2 + m^2\chi^2(a^2 - 2vv_2) \pm 2m^2\chi(sa - 2tv) = 0.$$

If χ is real, these two equations determine two branches of generally speaking complex values of $p^2 = -m^2g_{1,2}(\chi)$.[2)] When $\chi \sim 1$, the change of the electron mass is comparable with the damping: $\mathrm{Re}(p^2 + m^2) \sim \mathrm{Im}\, p^2$. The eigenspinors corresponding to these values of p^2 are given by

$$u_{1,2}(p,F) = (S - i\gamma V - \sigma T + i\gamma_5\gamma A)(1 \pm i\gamma_5\gamma n)w, \quad n_\mu = \frac{eF^*_{\mu\nu}p_\nu}{m^3\chi}, \tag{2}$$

$$n^2 = 1, \quad nV = 0,$$

(w is an arbitrary spinor) and correspond to two opposite natural orientations $\pm n_\mu$ of the electron spin in the field. Finally, the Green's function is

$$G(p,F) = -\frac{i}{2}(S - i\gamma V - \sigma T + i\gamma_5\gamma A)\sum_\pm \frac{1 \pm i\gamma_5\gamma n}{S^2+V^2+A^2 \pm 2(SA - 2T^*V, n)} \tag{3}$$

and has poles determined by Eq. (1), as well as a cut $m^2 \leq -p^2 < \infty$.

In conclusion, we present an explicit expression for the mass operator in

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[1)] The diagonality is the consequence of the commutativity of M with the operators $-i\partial_{1,2}$, $i(\partial_0 + \partial_3)$, and $(\gamma\pi)^2$, for which E_p is an eigenfunction with eigenvalues $p_{1,2}$, $p_- = p_0 - p_3$, and p^2 (the 3 axis is along $\vec{E} \times \vec{H}$).

[2)] The functions $g_{1,2}(\chi)$ are single-valued, for otherwise this would lead to additional degrees of freedom of the electron. Owing to the damping, we have $\mathrm{Im}\, g_{1,2} \leq 0$.

second order in the radiation field, and exact in the external crossed field:

$$M(p,F) = M_R^0 + \frac{\alpha}{2\sqrt{\pi}}\int_0^\infty \frac{du}{(1+u)^2}\left\{\left[2m + \frac{i\gamma p}{1+u} + \frac{ie^2\gamma FFp}{2m^4\chi^2}\left(1+\nu\frac{u-1}{1+u}\right)\right]f_1(z) - \right. \tag{4}$$

$$\left. - \left[\frac{ie\gamma_5\gamma Fp^*(2+u)}{m^2\chi(1+u)} + \frac{e\sigma F}{m\chi}\right]\left(\frac{\chi}{u}\right)^{1/3} f(z) - \frac{ie^2\gamma FFp}{m^4\chi^2}\frac{2+2u+u^2}{1+u}\left(\frac{\chi}{u}\right)^{2/3} f'(z)\right\},$$

where

$$z = (u/\chi)^{2/3}\left(1 - \frac{\nu-1}{u}\right),\quad \nu = -p^2/m^2,$$

f(z) is defined and tabulated in [3],

$$f_1(z) = \int_z^\infty dx(f(x) - 1/\sqrt{\pi}x),$$

and M_R^0 is the regularized value of the mass operator of the electron in vacuum, cf., e.g., [3]. On the mass shell $p^2 = -m^2$ the matrix element of this operator between free spinors determines, in second order, the amplitude of the elastic scattering of an electron in an intense crossed field, and in particular the field-dependent anomalous magnetic moment, cf. [3]. Expression (4) is important also for the determination of radiative corrections of higher orders.

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ASYMPTOTIC SOLUTION IN THE PROBLEM OF ANOMALOUS PLASMA RESISTANCE

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Assume that there exists in a plasma a constant homogeneous electric field E parallel to the magnetic field and strong enough to make it possible to neglect pair collisions. The electrons are then freely accelerated until the direction of their translational motion exceeds the threshold of excitation of oscillations of the acoustic type, after which an instability sets in and leads to deceleration of the electrons and to the appearance of the so-called "anomalous resistance." Within a time on the order of several reciprocal increments, the system returns to the threshold state and then remains continuously in this state. However, since the threshold of the current instability is proportional to the mean-squared plasma-particle velocity, which increases continuously under the influence of the electric field, the current will also increase with time.

After a sufficiently long time interval, the mean-squared plasma-particle velocity increases sufficiently to make the plasma "forget" its initial state, and the subsequent evolution of the system acquires a certain universal