second order in the radiation field, and exact in the external crossed field:

$$M(p,F) = M_R^0 + \frac{\alpha}{2\sqrt{\pi}}\int_0^\infty \frac{du}{(1+u)^2}\left\{\left[2m + \frac{i\gamma p}{1+u} + \frac{ie^2\gamma FFp}{2m^4\chi^2}\left(1+\nu\frac{u-1}{1+u}\right)\right]f_1(z) - \right. \tag{4}$$

$$\left. - \left[\frac{ie\gamma_5\gamma Fp^*(2+u)}{m^2\chi(1+u)} + \frac{e\sigma F}{m\chi}\right]\left(\frac{\chi}{u}\right)^{1/3} f(z) - \frac{ie^2\gamma FFp}{m^4\chi^2}\,\frac{2+2u+u^2}{1+u}\left(\frac{\chi}{u}\right)^{2/3} f'(z)\right\},$$

where

$$z = (u/\chi)^{2/3}\left(1 - \frac{\nu-1}{u}\right), \quad \nu = -p^2/m^2,$$

f(z) is defined and tabulated in [3],

$$f_1(z) = \int_z^\infty dx(f(x) - 1/\sqrt{\pi}x),$$

and M_R^0 is the regularized value of the mass operator of the electron in vacuum, cf., e.g., [3]. On the mass shell $p^2 = -m^2$ the matrix element of this operator between free spinors determines, in second order, the amplitude of the elastic scattering of an electron in an intense crossed field, and in particular the field-dependent anomalous magnetic moment, cf. [3]. Expression (4) is important also for the determination of radiative corrections of higher orders.

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ASYMPTOTIC SOLUTION IN THE PROBLEM OF ANOMALOUS PLASMA RESISTANCE

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Assume that there exists in a plasma a constant homogeneous electric field E parallel to the magnetic field and strong enough to make it possible to neglect pair collisions. The electrons are then freely accelerated until the direction of their translational motion exceeds the threshold of excitation of oscillations of the acoustic type, after which an instability sets in and leads to deceleration of the electrons and to the appearance of the so-called "anomalous resistance." Within a time on the order of several reciprocal increments, the system returns to the threshold state and then remains continuously in this state. However, since the threshold of the current instability is proportional to the mean-squared plasma-particle velocity, which increases continuously under the influence of the electric field, the current will also increase with time.

After a sufficiently long time interval, the mean-squared plasma-particle velocity increases sufficiently to make the plasma "forget" its initial state, and the subsequent evolution of the system acquires a certain universal character independent of the initial conditions. We determine in this paper the distribution functions of the charged particles and the spectrum of the oscillations in this regime, which we call the asymptotic regime. We show that actually there is no anomalous resistance in the asymptotic regime.

The problem can be solved on the basis of the quasilinear equations.[1] Formally, the presence of the asymptotic regime corresponds to the possibility of changing over in the quasilinear equations to self-similar variables [1]. It follows then from simple dimensionality considerations that the particle velocities should be measured in units of eEt/m, and the oscillation wave vectors in units of $m\omega_{pe}/eEt$, where e and m are respectively the charge and mass of the electron, and ω_{pe} is the electron plasma frequency.

Let us consider in greater detail the one-dimensional solution realized when the ion cyclotron frequency ω_{Hi} greatly exceeds the ion plasma frequency ω_{pi}. In this case the distribution functions $f_{e,i}$ of the electrons and ions, and the spectral density W of the electrostatic energy of the oscillations are given by

$$f_e(v, t) = \frac{mng_e(u)}{eEt}, \quad f_i(v, t) = \frac{mng_i(u)}{eEt}, \quad W(k, t) = m\omega_{pe}^4 t^2 U(q),$$

$$u = mv/eEt, \quad q = keEt/m\omega_{pe}$$

(n is the plasma density). Substituting these functions in the quasilinear equations written in the reference frame connected with the freely accelerated ions, we obtain

$$-\frac{d}{du}(u - 1 - \mu) g_e = \frac{d}{du} D(u) \frac{dg_e}{du} \quad , \tag{1}$$

$$-\frac{d}{du} ug_i = \mu^2 \frac{d}{du} D(u) \frac{dg_i}{du} \quad , \tag{2}$$

where D(u) is the quasilinear diffusion coefficient, and $\mu \equiv m/M$. Together with the condition for the vanishing of the oscillation increment

$$\frac{d}{du}(g_e + \mu g_i) = 0 \tag{3}$$

Eqs. (1) and (2) form a closed system that has the following solution[2]:

$$g_e = \frac{Cu}{u + \mu^2}, \quad g_i = \frac{C\mu(1 - u)}{u + \mu^2}, \quad D = \frac{u^2(1 - u)}{\mu^2} \text{ if } 0 < u < 1, \tag{4}$$

$$g_e = g_i = D = 0 \quad \text{if } u < 0 \text{ и } u > 1,$$

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[1] The role of the nonlinear effects decreases asymptotically in time, since the ratio of the oscillation energy to the particle kinetic energy, as will be shown subsequently, is proportional to t^{-1}.

[2] We shall henceforth assume for simplicity that $\mu \ll 1$, although it is also possible to obtain an exact solution valid for any ratio of the electron and ion masses.

where C is an arbitrary positive constant. It is possible to add to the functions $g_e(u)$ and $g_i(u)$ a certain number of freely accelerating electrons and ions, corresponding in the self-similar solution to a delta function at the point u = 1 for the electrons and u = 0 for the ions. Denoting the fractions of the freely accelerating particles by X_e and X_i, we see directly from the normalization conditions that $X_e + C = 1$ and $X_i + 2C\mu \ln\mu^{-1} = 1$.

Knowing the functions g_e and g_i, we can easily write the dispersion relation

$$\epsilon(q, \omega) = 1 - \frac{1-C}{(\omega-q)^2} - \frac{\mu}{\omega^2} + \frac{C}{\omega q} - \frac{C}{(\omega-q)q},$$

where the frequency is measured in units of ω_{pe}. The function $\varepsilon(q, \omega)$ should satisfy the following two requirements: (1) all the oscillations must be stable; (2) there should exist oscillations with all the phase velocities in the interval (0, 1). From these conditions, we can determine uniquely the constant C (which turns out to equal $2\mu^{1/2}$), and by the same token the distribution functions $g_{e,i}$. As to the oscillation spectrum it can be determined from the formula

$$U(q) = \frac{eE^3}{8\pi^2 m^2 \omega_{pe}^4} \left| \frac{d\omega(q)}{dq} - \frac{\omega(q)}{q} \right| D\left[\frac{\omega(q)}{q}\right]$$

This solves the problem completely.

Let us list the main qualitative features of the obtained solution: (1) almost all the electrons and ions are freely accelerated by the electric field ($X_e = 1 - 2\mu^{1/2}$, $X_i = 1 - 4\mu^{3/2}\ln\mu^{-1}$); (2) in spite of this, the system is at the instability threshold, a fact ensured by the presence of small groups of electrons and ions with velocity "spreads"; (3) there are present very "hot" ions with energies μ^{-1} times larger than the energy of the freely accelerated electrons (the relative concentration of such ions is equal approximately to $\mu^{3/2}$); (4) in the asymptotic regime, the energy of the Langmuir oscillations is comparable with the energy of the acoustic oscillations.

The obtained solution holds for very strong magnetic fields ($\omega_{He} \gg \mu^{-1/2}\omega_{pe}$). In weaker magnetic fields ($\omega_{pe} \ll \omega_{He} \ll \mu^{-1/2}\omega_{pe}$) the one-dimensional solution becomes unstable against excitation of "oblique" waves, and the spectrum of the oscillations becomes essentially three-dimensional. An analysis shows that in this case the system is stabilized as a result of the "smearing" of the ion distribution function in the transverse direction; as to the electrons, they are accelerated, as before, in a compact group. Only in still weaker fields, when $\omega_{He} \ll \omega_{pe}$, does the thermal spread of the electrons become of the order of their current velocity. It must be particularly emphasized that the Langmuir oscillations play an important role in the formation of the solution for arbitrary magnetic fields.

All the foregoing results can be readily extended to the case when the electric field varies in time (but does not pass through zero!). To this end it is necessary merely to introduce the new self-similar variables $u = mv/e\int_0^t E dt'$ and $q = ke\int_0^t E dt'/m\omega_{pe}$.

A characteristic feature of the described solution is the presence of runaway electrons. The situation changes radically if the current flows across the magnetic field[3], when the runaway electrons disappear completely. On the other hand, if the effective frequency of the collisions between the electrons and the oscillations does not exceed ω_{He} (as is usually the case in experiments with shock waves), then in the coordinate frame that moves relative to the ions with a drift velocity cE/H, the electron distribution function turns out to be in general axially symmetrical with respect to H, and the current velocity of the electrons becomes much smaller than the thermal velocity. In order to illustrate the last statement, we consider a model problem, in which it is assumed that the oscillations are excited only in the direction of the electron current. Such a problem is described by equations analogous to (1) - (3) and are just as easy to solve. Calculations have shown that the current velocity of the electrons $\bar{u}$ amounts to $2.1\mu^{2/5}$ of their mean-squared velocity $(\overline{u^2})^{1/2}$. It must be noted, incidentally, that now, unlike in the case considered above, the allowance for the "oblique" waves may lead to an appreciable change of the solution. Thus, an approximate alanlsis made in [1] with allowance for the "oblique" waves has shown that $\bar{u} \sim \mu^{1/4}(\overline{u^2})^{1/2}$.

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[3]Even one so weak that it affects neither the dispersion of the oscillations nor the motion of the ions.