region, owing to excitation of the atoms or molecules and to electrostriction. However, a comparison of characteristic times of development and duration of the laser pulse shows that these mechanisms cannot make a noticeable contribution to the observed effect.

The foregoing experimental observation of stimulated Compton absorption in a plasma confirms the theoretical assumption [1 - 3] that this effect can play an important role in the heating of a plasma by electromagnetic radiation. If the radiation intensity is high enough, then at low plasma densities and at high temperatures the SCS may prevail over the classical and bremsstrahlung absorption, which furthermore are reduced by the nonlinear effects in a strong field [5, 6]. However, the physical nature of this type of absorption gives grounds for hoping to obtain a sufficiently effective contribution of the radiation energy to the plasma, provided only the width of the emission spectrum $\Delta\nu$ is comparable with the emission frequency ν [3]. This raises the problem of producing special high-power sources of coherent radiation.

Our experimental results do not agree with the hypothesis in [1] that the electron-heating mechanism in question can play an important role during the development stage of breakdown at optical frequencies. In particular, the estimate of the magnitude of this effect under the conditions of experiments on breakdown by picosecond pulses [7] shows that it is not decisive in the production of the laser spark.

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PARTICLE PRODUCTION IN COSMOLOGY

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According to rough semiquantitative estimates, the production of elementary particles near a singularity in an anisotropic cosmological model is capable of resulting in an energy density sufficient for isotropization of the expansion. If the aforementioned estimates are correct then, when account is taken of particle production, the power-law asymptotic form of a singularity of the Kasner type turns out to be internally contradictory as applied to cosmology, and all that can remain is the degenerate case of a fictitious singularity with exponents 1, 0, and 0 or an isotropic Friedmann singularity. Particle production may turn out to be of importance for the explanation of the presently observed ratio of the total number of particles (mainly photons) to the number of baryons.

The question of particle production was considered in the cosmological problem as applied to an isotropic (Friedmann) solution in a paper by Parker.

In the present paper we consider a singularity of the Kasner type (see [2])

$$ds^2 = c^2dt^2 - t^{2p_1}d\xi^2 - t^{2p_2}d\eta^2 - t^{2p_3}d\zeta^2. \tag{1a}$$

The role of (1a) as a prototype of the most general solution with a singularity, and for the description of the initial state of the evolution of the universe, was considered in [3 - 5].

From the point of view of a local Newtonian observer in a space with coordinates

$$x = t^{p_1}\xi, \quad y = t^{p_2}\eta, \quad z = t^{p_3}\zeta$$

we have a gravitational potential (see [6, 7])

$$\phi = -\frac{1}{2t^2}[p_1(p_1 - 1)x^2 + p_2(p_2 - 1)y^2 + p_3(p_3 - 1)z^2], \qquad \Delta\phi = 0. \tag{1b}$$

Let us assume that $p_1 < p_2 < p_3$, so that $p_1 < 0$ and the potential along the x axis as a maximum at the origin, $\phi = -kx^2$, $k > 0$. We can assume that such a potential is favorable for the production of pairs moving in opposite directions along the x axis with an ever increasing velocity. In analogy with the production of charged pairs in a static electric field (cf. e.g., [8]) we find the half-width of the barrier r from the condition

$$\phi(r) = \phi(0) - c^2, \; r = c/\sqrt{k}. \tag{2}$$

If the width is smaller than the Compton wavelength of the particle, then we can expect the production probability to be independent of the rest mass of the particle, and to be given by an expression that follows from dimensionality considerations:

$$\frac{dn}{dt}\,\frac{1}{\mathrm{cm}^3\,\mathrm{sec}} = k^2c^{-3} = \frac{p_1^2(p_1 - 1)^2}{t^4c^3}. \tag{3}$$

An analysis based on the method of [1] leads to a similar result. It is possible to state the problem rigorously only if a static metric is specified at $t = \pm\infty$ and the singularity is replaced by continuous and finite expressions for the metric coefficients in the region $-t_0 < t < t_0$. A static metric is necessary at $t = -\infty$ for an unambiguous definition of the vacuum and at $t = +\infty$ for an unambiguous determination of the number of produced particle. The elimination of the singularities in the region $|t| < t_0$ is necessary in order to obtain a single-valued solution.

As shown in [1] and [9], particle production is determined by the ratio $b(t = +\infty)/b(t = -\infty)$ of the quantity $b(t)$ satisfying an equation of the type

$$\ddot{b} + \omega^2(t)\,b = 0, \tag{4}$$

where ω is the frequency of the given proper mode of the field describing the particles.

It seems paradoxical that the particles are produced separated by a distance exceeding ct (see Formula (2)).

This paradox is the consequence of describing classically in the language of particles having definite coordinates, phenomena that are essentially linked inseparably with the wave properties of matter, namely, the passage under a potential barrier.

An analogous paradox arises also in the production of charged particles by an electrostatic field E; the classical trajectories are given by the expression

$$x_{\pm} = x_0 \pm \sqrt{\left(\frac{mc^2}{eE}\right)^2 + c^2(t - t_0)^2} .$$

The sum of the momenta p_+ and p_-, where

$$p = mc\,\beta/\sqrt{1-\beta^2}, \quad \beta = \frac{1}{c}\frac{dx}{dt}$$

is $p_+(t) + p_-(t) = 0$.

This property, just like the equation for the trajectories, is Lorentz-invariant. In any system, however, the interval between the world points of electron and positron production is space-like. Although the production of the two particles of the pairs is interrelated, since e^+ or e^- cannot be produced separately, a causal connection is impossible in the classical description. Let us turn now to the gravitation problem.

In the case of a spatially homogeneous problem, the spatial function of the wave field is given by the plane wave $\exp(k_1\xi + k_2\eta + k_3\zeta)$, so that the wave vector and the frequency (at m = 0) are given by the expression

$$\omega^2/c^2 = |\,k\,|^2 = k_1^2 t^{-2p_1} + k_2^2 t^{-2p_2} + k_3^2 t^{-2p_3} . \qquad (5)$$

An equation of the type (4) solves the problem of the behavior of the classical wave field in the specified time-dependent metric.

The number of quanta is an adiabatic invariant [10] of the classical field. Violation of adiabatic invariance in the classical problem, $|b_+|/|b_-| > 1$, in accord with the corresponding principle, describes the production of a pair of particles in vacuum in quantum field theory.

Approximate solution of (4) are (we put $\omega(t = \infty) = \omega(t = -\infty)$ and $n = b_+^2 - b_-^2/b_-^2$)

$$n \cong \int_{-\infty}^{+\infty} \frac{d\ln\omega}{dt} e^{2i\int\omega dt} dt = \frac{1}{2i}\int \omega^{-2}\frac{d\omega}{dt} e^{2i\int\omega dt} dt \cong \frac{1}{i}\int \frac{d}{\sqrt{\omega}}\frac{d^2\sqrt{\omega}}{dt^2} e^{2i\int\omega dt} . \qquad (6)$$

The answer depends mainly on the vicinity of the singularity. It can be roughly assumed that the production takes place near t_0 and that particles with $\omega(t_0) < t_0^{-1}$ are produced with a probability on the order of 1. The density of such particles is in this case

$$n\frac{1}{\text{cm}^3} = k^3 = \frac{\omega^3}{c^3} = \frac{1}{c^3 t_0^3} . \qquad (7)$$

This expression coincides in order of magnitude with that obtained from (3) by substituting $n(t_0) \simeq t_0(dn/dt)|_{t=t_0}$. The difference lies only in a factor that depends on p_1 and vanishes when $p_1 = 0$. This difference is not surprising. The metric with p_1 has only a fictitious singularity, but in the formulation of the problem (with a change over from contraction to expansion near the singularity) such a transition introduces a nonzero curvature. The average particle energy at the instant of production, corresponding to (7), is of the order of $h\omega \sim h/t_0$, so that the energy density is $\varepsilon_0 \sim h/c^3t_0^4$.

Let us stop to discuss the place occupied by spontaneous production of particles in a gravitational field in general relativity theory (GRT).

The classical equations

$$R_{ik} - \frac{1}{2} g_{ik}R = \kappa T_{ik} \tag{8}$$

are not compatible with the production of particles, since they lead to the identity $T^k_{i,k} \equiv 0$. Let the initial state be vacuum, and let T_{ik} and its derivative be equal to zero on the hypersurface t = const or $t = -\infty$. It then follows from $T^k_{i,k} = 0$ that the vacuum is always conserved.

Consequently, particle production is of necessity connected with corrections to the GRT, or more accurately with quantum corrections, since $\varepsilon \sim h$.

The quantum corrections to the GRT equations were considered in several papers, on the basis of the classical equations [11, 12], and most recently [13], as well within the framework of a new approach [14] to the derivation of the GRT equations. The corrections considered there were "real" corrections of the order of the square of the curvature and of higher order, i.e., the nonlinear change in the elasticity of the vacuum.

Particle production is an "imaginary" correction, having the meaning of "viscosity" of the vacuum.

Let us turn to the cosmological consequences of particle production. The period of intense production at t_0 is followed by expansion with a decrease of the energy density, following the law

$$\epsilon = \epsilon_0\left(\frac{t}{t_0}\right)^{-\alpha}, \qquad \frac{4}{3} \geqslant \alpha \geqslant 1 - |p_1|, \tag{9}$$

where the exponent α depends on the assumption made concerning the interaction between the particles [15].

Let us find now the instant t_1 when this energy density is sufficient to exert a gravitational action on the metric and to transform the Kasner solution into a Friedmann solution. From the condition

$$\epsilon(t_1) \approx c^2/Gt^2, \tag{10}$$

we obtain

$$t_1 = (c^5G^{-1}\hbar^{-1}t_0^{4-\alpha})^{1/2-\alpha} = (t_0^{4-\alpha}t_p^{-2})^{1/2-\alpha}, \tag{11}$$

where $t_p = 10^{-45}$ sec. Consequently, if we start the calculation with $t_0 \sim t_p$, then the transition to the Friedmann solution occurs practically at the same instant, and the Kasner solution terminates here in suicide. This leaves open the question of the situation in the case of the exponents 1, 0, and 0. Apparently it is necessary to make a deeper analysis of the singularities. Within the framework of the Friedmann model, according to Parker [1], no massless particles are produced at $p = \varepsilon/3$ and $a(t) \sim \sqrt{t}$. Let us consider an initial state in the form of a cold world with an extremely rigid [16] equation of state (m is the baryon rest mass)

$$p = \epsilon = n^2 m^4 c^5 \hbar^{-3}, \qquad a(t) \sim t^{1/3}.$$

The spontaneous production of (pairs of) particles in such a metric leads to a growth of the entropy. Stipulating $t_0 = t_p$, we obtain in order of magnitude, the dimensionless entropy

$$s \sim \sqrt{\hbar c / G M^2} \sim 10^{18}$$

in place of the observed $s \sim \gamma/n_b \sim 10^8$, see [6, 17]. It is not excluded that some modification of the equation of state yields the correct value of s.

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